Teachers' Handbook

Volume 1: Mathematical Literacy

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ACKNOWLEDGMENTS

MATHEMATICAL LITERACY - A HANDBOOK FOR TEACHERS

INDEX

1. INTRODUCTION

The National Policy on Education 1986 proposed that "Mathematics should be visualized as the vehicle to train a child, to think, reason, analyze and to articulate logically". There is a shift in treating Mathematics Education as an instrument for the National development to the development of child's abilities. This has been carried forward further in the National Curriculum Framework 2005 wherein child's active engagement in mathematics learning involving "inquiry, exploration, questioning, debates, application and reflection leading to theory building and creation ideas/positions in Mathematics" is envisaged.(1)

1.1 Objectives of teaching Mathematics:

The objective of the school Mathematics curriculum at the elementary and secondary level is to provide students opportunities to be equipped with important Mathematics concepts needed for better educational, professional and social choices. It empowers students to investigate, to make sense of and to construct mathematical meanings from new situations. By learning Mathematics, students must be able to develop logical thinking, reasoning and to find out Mathematics in all walks of life including Art. They should not find the subject in isolation with other subjects or life-rather be able to make use of their learning in day to day life.

1.2 Curricular Expectations:

According to the Learning outcomes document prepared by NCERT and recommended by the CBSE for all its affiliated schools, the curricular expectations from Mathematics in **upper primary classes (VI-VIII)** are as under: (2)

Children are expected to:

a) move from concrete ideas of numbers to number sense

b) see relationships between numbers and look for patterns in relationships

c) understand and apply concepts related to variables, expressions, equations, identities, etc.

d) use arithmetic and algebra to solve real- life problems and pose meaningful problems

e) develop an aesthetic sense by discovering symmetries in shapes like triangles, circles and quadrilaterals

f) identify space as a region enclosed within boundaries of a shape

g) develop spatial understandings in terms of perimeter, area and volume and use them to solve day-to-day life problems

h) learn to provide reasoning and convincing arguments to justify her/his own conclusions in a mathematical context

i) collect, represent (graphically and in tables) and interpret data/information from her/his life experiences.

1.3 Learning Outcomes:

The learning outcomes based on curricular expectations are set for each class for the elementary classes.(2)

VI Learning Outcomes

The learner-

- solves problems involving large numbers by applying appropriate operations (addition, subtraction, multiplication and division)

- recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc.

- applies HCF or LCM in a particular situation

- solves problems involving addition and subtraction of integers.

- uses fractions and decimals in different situations which involve money, length, temperature etc. For example, 7½ metres of cloth, distance between two places is 112.5 km etc.

- solves problems on daily life situations involving addition and subtraction of fractions / decimals

- uses variable with different operations to generalise a given situation. e.g., perimeter of a rectangle with sides x units and 3 units is $2(x+3)$ units

- compares quantities using ratios in different situations. e.g., the ratio of girls to boys in a particular class is 3:2

- uses unitary method in solving various word problems. For example, if the cost of a dozen notebooks is given she finds the cost of 7 notebooks by first finding the cost of 1 note book

- describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc., with the help of examples in surroundings

- demonstrates an understanding of angles by
  - identifying examples of angles in the surroundings
  - classifying angles according to their measure
  - estimating the measure of angles using 45°, 90° and 180° as reference angles

- demonstrates an understanding of line symmetry by
  - identifying symmetrical 2-Dimensional (2-D) shapes which are symmetrical along one or more lines
  - creating symmetrical 2-D shapes

- classifies triangles into different groups/ types on the basis of their angles and sides.
  - e.g. scalene, isosceles or equilateral on the basis of sides, etc.

- classifies quadrilaterals into different groups/types on the basis of their sides/angles

- identifies various (3-D) objects like sphere, cube, cuboid, cylinder, cone from the surroundings

- describes and provides examples of edges, vertices and faces of 3-D objects

- finds out the perimeter and area of rectangular objects in the surroundings like floor of the classroom, surfaces of a chalk box etc.

- arranges given/collected information such as expenditure on different items in a family in the last six months, in the form of a table, pictograph and bar graph and interprets them.

VII Learning Outcomes

The learner-

- multiplies/divides two integers
- interprets the division and multiplication of fractions.
- for example interprets $\frac{2}{3} \times \frac{4}{5}$ as $\frac{2}{3}$ of $\frac{4}{5}$ Also $\frac{1}{2} \div \frac{1}{4}$ is interpreted as how many $\frac{1}{4}$ make $\frac{1}{2}$?
- uses algorithms to multiply and divide fractions/decimals.
- solves problems related to daily life situations involving rational numbers
- uses exponential form of numbers to simplify problems involving multiplication and division of large numbers.
- represents daily life situations in the form of a simple equation and solves it
- adds/subtracts algebraic expressions
- distinguishes quantities that are in proportion. For example, tells that 15, 45, 40, 120 are in proportion as $\frac{15}{45}$ is the same as $\frac{40}{120}$
- solves problems related to conversion of percentage to fraction and decimal and vice versa
- calculates profit/loss percent and rate percent in simple interest
- classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given.

- verifies the properties of various pairs of angles formed when a transversal cuts two lines
- finds unknown angle of a triangle when its two angles are known
- explains congruency of triangles on the basis of the information given about them like (SSS, SAS, ASA, RHS)
- using ruler and a pair of compasses constructs, a line parallel to a given line from a point outside it and constructs triangles on the basis of congruency
- finds out approximate area of closed shapes by using unit square grid/ graph sheet
- calculates areas of the regions enclosed in a rectangle and a square
- finds various representative values for simple data from her/his daily life contexts like mean, median and mode
- recognises variability in real-life situation such as, variations in the height of students in her class and uncertainty in happening of events like throwing a coin
- interprets data using bar graph such as consumption of electricity is more in winters than summer, runs scored by a team in first 10 overs etc.

VIII Learning Outcomes

The learner-

- generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns

- finds out as many rational numbers as possible between two given rational numbers.

- proves divisibility rules of 2, 3,4, 5, 6, 9 and 11

- finds squares, cubes and square roots and cube roots of numbers using different methods.

- solves problems with integral exponents.

- solves puzzles and daily life problems using variables.

- multiplies algebraic expressions. e.g expands $(2x-5)(3x^2+7)$.

- uses various algebraic identities in solving problems of daily life

- applies the concept of percent in profit and loss situation in finding discount, VAT and compound interest. e.g., calculates discount percent when marked price and actual discount are given or finds profit percent when cost price and profit in a transaction are given.

- solves problems based on direct and inverse proportions

- solves problems related to angles of a quadrilateral using angle sum property

- verifies properties of parallelograms and establishes the relationship between them through reasoning

- represents 3D shapes on a plane surface such as a sheet of paper, blackboard etc.

- verifies Euler's relation through pattern

- constructs different quadrilaterals using compasses and straight edge.

- estimates the area of shapes like trapezium and other polygons by using square grid/ graph sheet and verifies using formulas.

- finds the area of a polygon.

- finds surface area and volume of cuboidal and cylindrical object.

- draws and interprets bar charts and pie charts.

- makes hypotheses on chances of future events on the basis of its earlier occurrences or available data such as, after repeated throws of dice and coins

1.4 Present status of Mathematics Learning

Various educational surveys and achievement data over the years show that the learning achievements of children in various subjects-especially Mathematics- are not up to the expected levels despite all the efforts made by states in this direction. A summary of the National Achievement Survey results for Mathematics in CBSE affiliated schools for Class X level (marked as a state) in the academic year 2017-18 is given below: (3)

Skill-wise performance: (ITEM % CORRECT)

Subject	Skill 1 (% correct)	Skill 2 (% correct)	Skill 3 (% correct)
Mathematics	56	51	48

Skill 1	**Remembering :** Recognizing or recalling knowledge from memory.
Skill 2	**Understanding:** Constructing meaning from different types of functions be they written or graphic messages.
Skill 3	**Applying:** Carrying out or using a procedure through executing, or implementing.

Performance of Students: CONTENT WISE

Subjects	Content domain	State average (percent correct)	National average (percent correct)	Significant difference
Mathematics	Mathematics (overall)	52	34	⬆
	Algebra	53	35	⬆
	Geometry	54	35	⬆
	Mensuration	54	34	⬆
	Trigonometry	60	37	⬆
	Coordinate Geometry	44	29	⬆
	Number System	44	30	⬆
	Statistics	42	33	⬆
	Probability	57	35	⬆

Sample questions and the number of students answered are indicated below. The percentage of students who answered correctly is given in red.

If $ax^2+bx+c=0$ has equal roots, then c is equal to

1. $\dfrac{-b}{2a}$

2. $\dfrac{b}{2a}$

3. $\dfrac{-b^2}{4a}$

4. $\dfrac{b^2}{4a}$

Skill : Skill 3 item
Content Domain : Algebra

% Answers:
1. 30.0%
2. 28.1%
3. 19.5%
4. 19.8%

If the nth term of an A.P. is (3n+1), then its 30th term is

1. 31

2. 91

3. 90

4. 94

Skill : Skill 2 item
Content Domain : Algebra

% Answers:
1. 25.8%
2. 32.0%
3. 27.8%
4. 8.9%

AOB is a sector of circle of radius 4 cm subtending an angle of 45^0 at the centre 'O' of the circle. Area of the sector, in cm^2, is

1. ð

2. 2ð

3. 3ð

4. 4ð

Skill : Skill 1 item
Content Domain : Mensuration

% Answers:
1. 17.4%
2. 49.6%
3. 11.2%
4. 17.5%

If $\triangle ABC \cong \triangle PQR$, then which of the following is true?

1. $B \leftrightarrow R$

2. $C \leftrightarrow Q$

Skill : Skill 1 item
Content Domain : Geometry

3. $A \leftrightarrow R$

% Answers:
1. 11.1%
2. 13.2%
1. $A \leftrightarrow P$
3. 15.8%
4. 56.5%

If $\tan\theta = \dfrac{3}{4}$, then $\cot\theta$ is equal to

1. $\dfrac{4}{3}$

2. $\dfrac{3}{7}$

Skill : Skill 1 item
Content Domain : Trigonometry

3. $\dfrac{4}{7}$

% Answers:
1. 71.1%
2. 11.4%
3. 9.2%
4. $\dfrac{7}{4}$
4. 5.5%

From the above information, one can see that students can perform better in the questions which are purely based on remembering. It can be concluded that the curricular expectations as defined by the NCERT are not being met in most of the cases. Though the performance of students in the CBSE affiliated schools is above the national average, efforts have to be made to improve the learning outcomes.

1.5 Objectives of the Manual:

The Manual gives an overview of Mathematical Literacy with a special focus on PISA with reference to the content, process and context involved. It is expected that teachers will make use of this manual for integrating Assessment in the classroom transactions. Pedagogical strategies including assessment for learning are discussed in detail with a special emphasis on connecting mathematics to daily life, an inter-disciplinary approach to Mathematics learning and mathematisation. We hope this manual will help in giving a direction to the teacher to improve the competencies of children in mathematics so that they can face the challenges of day to day life in the 21st Century.

Note to Teachers: Teachers must thoroughly understand the curricular expectations and learning outcomes of each stage. All the classroom transactions should be planned and carried out by keeping these in mind rather than simply covering the syllabus or preparing the students for some examinations.

2. INTRODUCTION TO PISA

2.1 What is PISA ?

The Programme for International Student Assessment (PISA), a project of member countries of the OECD (Organization for Economic Co-operation and Development), is a triennial international assessment which aims to evaluate education systems worldwide by testing the skills and knowledge of 15-year-old students who are nearing the end of their compulsory education. PISA is designed to assess how well they can apply what they learn in school to real-life situations. Over 90 countries have participated in the assessment so far, which has taken place at three-year intervals since 2000.(4)

2.2 Why PISA ?

PISA, a competency-based assessment, has been designed to assist governments in monitoring the outcomes of education systems in terms of student achievement on a regular basis and within an internationally accepted common framework. In other words, it will allow them to compare how students in their countries are performing on a set of common tasks compared to students in other countries. In this way, PISA helps governments to not only understand, but also to enhance, the

Why PISA for India?

- PISA—will help to introduce competency-based examination reforms in the school system.

- Will discourage rote learning

- Lead to recognition and acceptability of Indian students and prepare them for the global economy of 21st Century.

effectiveness of their educational systems and to learn from other countries' practices. It can help policymakers use the results of PISA to make decisions about education, to inform their teaching and set new targets.

Given the rapid pace with which changes are happening around us, we need our education to prepare us for the times to come. **Critical Thinking, Problem Solving and Conceptual understanding** must become the key parameters of learning. The relevance of information is losing value but the ways and means of processing that information are progressively becoming more relevant.

Our current education leaves our children far behind in all these aspects and in that light PISA is a welcome change. We need to look at PISA as an intervention that will not only make our education more relevant for current times but also make it Future Proof. Activities and concepts in the classrooms need to engage the mind, inspire and become contextual. Evaluation needs to be re-modeled to incorporate similar thinking. In that light PISA is a very welcome intervention for both educators and learners.

2.3 PISA Proceduree

Every three years, students complete an assessment that includes items, testing **(Reading Literacy, Mathematical Literacy and Scientific Literacy)**. In each cycle of PISA, **one of the cognitive areas is the main focus of the assessment, with most of the items focusing on this area and fewer items** on the other two areas. In addition to these three, students are tested in an innovative domain such as collaborative problem solving in 2015 and global

The Indian Plan for PISA

- CBSE & NCERT – part of the process and activities leading to the actual test.

- Field Trial(FT) – to be conducted in April 2020.

- 25 schools x 36 students each =900 students to be assessed in FT.

- PISA 2021 – officially called Main Survey--to be conducted in April 2021.

competence in 2018. Students also complete an **extensive background questionnaire**, and school Principals complete **a survey describing the context of education** at their school, including the level of resources in the school, qualifications of staff and teacher morale. The data collected from the assessment and background questionnaires are analyzed and the results are published a year after the assessment. These studies enable the participating countries to benchmark their students against similar samples from students of other countries.

2.4 PISA 2021

- India shall be participating for the second time, the first being in 2009.
- 36 OECD member countries and over 50 non-members are expected to participate.
- Each student, selected on the basis of random sampling, will be tested on any two out of the three domains, viz. Reading Literacy, Mathematical Literacy and Scientific Literacy.

PISA 2021

5250 students (150 schools x 35 students) will be assessed on the following subjects in the paper-based assessment:-

- Mathmatics and Science (33% students)
- Mathmatics and Reading (33% students)
- Reading and Science (33%students)

Assessment Goals:

- ✓ To evaluate outcomes of learning.
- ✓ To assess how well students can apply what they learn in school to real-life situations.
- ✓ To show what 15 year-olds have learnt inside and outside a classroom.
- ✓ To measure literacy in terms of knowledge, skills and competencies.

The report of the findings from PISA focuses on issues such as:

- ✓ How well are young adults prepared to meet the challenges of the future ?
- ✓ Can they analyze, reason and communicate their ideas effectively ?
- ✓ What skills do they possess that will facilitate their capacity to adapt to rapid societal change ?
- ✓ Are some ways of organizing schools or school learning more effective than others ?
- ✓ How does the quality of school resources influence student outcomes ?
- ✓ What educational structures and practices maximize the opportunities of students from disadvantaged backgrounds ?
- ✓ How equitable is the provision of education within a country or across countries ?

2.5 Principles of Testing in PISA

I. Paper and Pen Assessment

- **Cognitive Assessment c**overs three domains: R**eading Literacy, Mathematical Literacy and Scientific Literacy.**

- The assessment of cross-curriculum competencies is an integral part of PISA.
- Emphasis is placed on the mastery of processes, the understanding of concepts and the ability to function in various situations within each domain.
- Thus, PISA test is different:
  - ✓ Focus is on understanding
  - ✓ Proper reading
  - ✓ No guesswork
  - ✓ Answers may be related to previous answers
  - ✓ Participating students complete a two-hour paper and-pen assessment.

II. Context questionnaire

To gather contextual information, PISA asks students and the principals of their schools to respond to questionnaires. These take about 35 and 45 minutes, respectively, to complete.

The questionnaires seek information about:

- Students and their family backgrounds, including their economic, social and cultural capital.
- Aspects of students' lives, such as their attitudes towards learning, their habits and life in and outsides of schools and their family environment.
- Aspects of schools, such as the quality of the schools' human and material resources, public and private management and funding, decision-making processes, staffing practices and the school's curricular emphasis and extracurricular activities offered.
- Context of instruction, including institutional structures and types, class size, classroom and school climate and science activities in class.
- Aspects of learning, including students' interest, motivation and engagement.

Target Group

- Children in the age group 15 years 3 months to 16 years 2 months attending any educational institution in the country (selected region), including public, private, aided, international schools.
- Open Schools students are not eligible.
- As test will be held in April 2021, students born between February 2005 to January 2006 are considered.

2.6 The Challenge Ahead

- 2021--PISA will help reveal where India stands globally as far as learning outcomes are concerned.
- The participation in PISA 2021 would indicate the health of the education system and would motivate other states in the subsequent cycles. This will lead to improvement in the learning levels of the children and enhance the quality of education in the country.
- The Challenge before the teaching community is to collaborate, train and brace our students for PISA 2021.

3. MATHEMATICAL LITERACY IN PISA

Mathematical Literacy can be defined as an "individual's capacity to identify and understand the role that Mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen".(5) All citizens need to be mathematically literate in order to make an effective contribution to society. In our country, many students after completing their secondary education may not be formally studying Mathematics for the rest of their lives. Hence they must be able to make use of school Mathematics, in the situations they are likely to encounter in their daily lives. This will help them to lead a successful adult life.

3.1 Mathematical Literacy Framework in PISA:

Mathematical Literacy includes making mathematical deductions and applying mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It helps people to identify and understand the role that mathematics plays in the world, and to make the well-founded judgments and decisions required in life.

As such, the exercises used in PISA primarily focus on the daily practical applications of mathematical knowledge in different real contexts. By contrast, Mathematics in which an explicit mathematical problem is formulated without a real context, is unlikely to be the subject of PISA mathematical exercises. For PISA, the content, context and processes that are focused upon are given below:

Content areas:
- Quantity
- Space and Shape
- Change and Relationships
- Uncertainty and Data

Contexts:
- Personal
- Occupational
- Societal
- Scientific

Processes:
- formulating situations mathematically
- employing mathematical concepts, facts, procedures and reasoning
- interpreting, applying and evaluating mathematical outcomes

21st Century Skills
- Critical Thinking
- Creativity
- Research and inquiry
- Self-direction, initiative, and persistence
- Information use
- Systems thinking
- Communication
- Reflection

Contexts
- Personal
- Occupational
- Societal
- Scientific
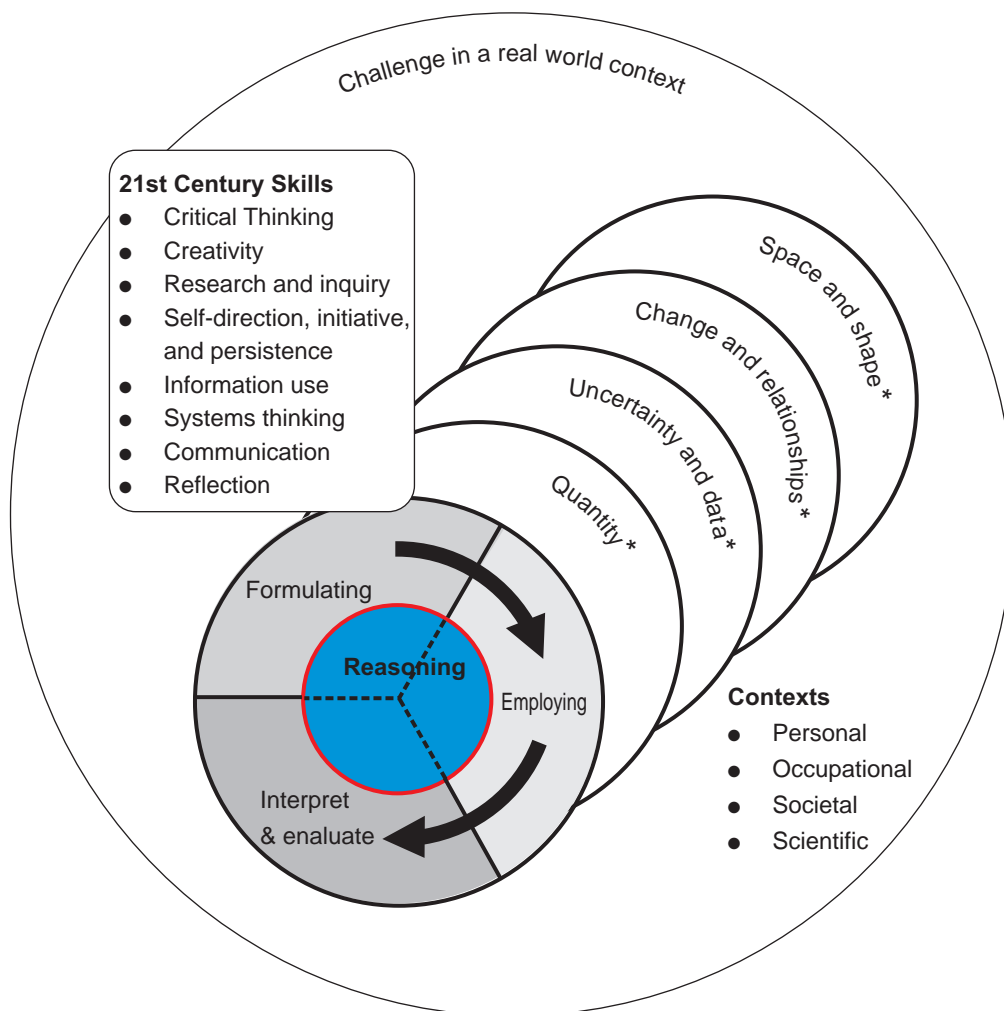
Diagrammatic representation of the content, context and processes for Mathematical Literacy

3.2   Processes involved in Mathematical thinking:

Mathematical literacy emphasizes on three processes namely, **formulate, employ and interpret.**

 Hence we must equip students in

1)   **formulating** situations mathematically

2)   **employing** mathematical concepts, facts, procedures and reasoning

3)   **interpreting, applying and evaluating** mathematical outcomes.

Teachers should see, how effectively students are able to engage in each of these processes. Mathematical processes and the underlying mathematical capabilities can be depicted as under in the context of a real-life problem:
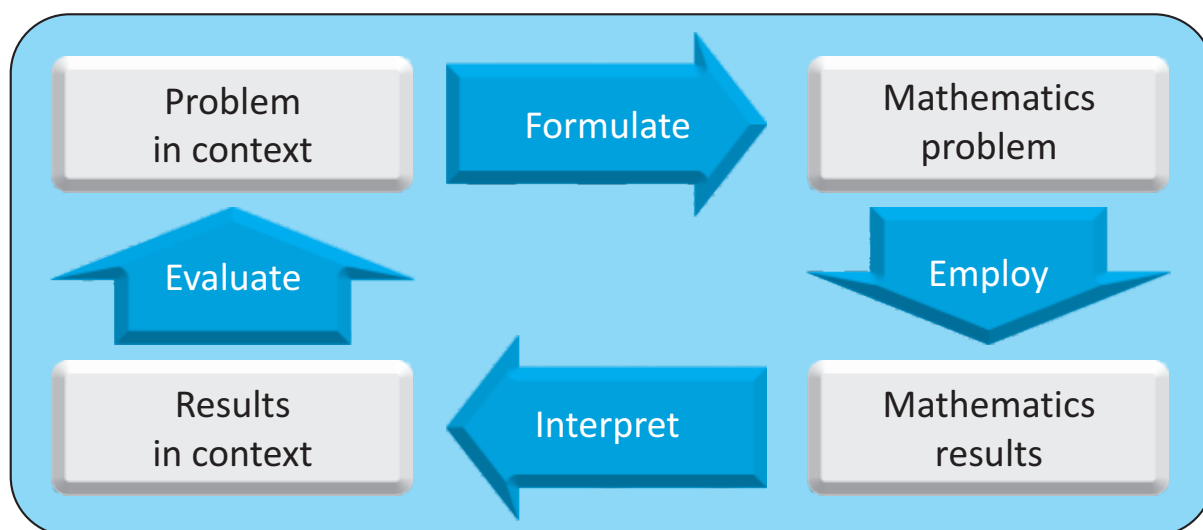
The processes outlined in the model are the key components of solving a real world problem, where the **starting point is the problem in its context**, not Mathematics. Let us understand the processes in detail.

1. **Formulating situations mathematically :**

    This involves identifying how to apply and use Mathematics to the problem being posed in the real world—it includes being able to take a situation and transform it into a form amenable to mathematical treatment. This process indicates how effectively students are able to recognize and identify opportunities to use Mathematics in problem situations and then provide the necessary mathematical structure needed to formulate that contextualized problem into a mathematical form.

    Specifically, this process of *formulating* situations mathematically includes activities such as the following: (5)

    a. **identifying the mathematical aspects of a problem** situated in a real-world context and identifying significant variables

    b. **recognizing mathematical structure** (including regularities, relationships and patterns) in problems or in situations

    c. **simplifying a situation or problem** in order to make it amenable to mathematical analysis

    d. **identifying constraints and assumptions** behind any mathematical modeling and simplifications

    e. **representing a situation mathematically**, using appropriate variables, symbols, diagrams and standard models

    f. **representing a problem in a different way**, including organising it according to mathematical concepts and making appropriate assumptions

    g. **understanding and explaining the relationships** between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically

    h. **translating a problem into mathematical language** or a representation

    I. **recognizing aspects of a problem** that correspond with known problems or mathematical concepts, facts or procedures

Let us take an example of a PISA sample question which focuses on the ability - 'Formulate'

Example:

This is the plan of the apartment that Anil's parents want to purchase from a builder. (7)

Scale:
1 cm represents 1 m
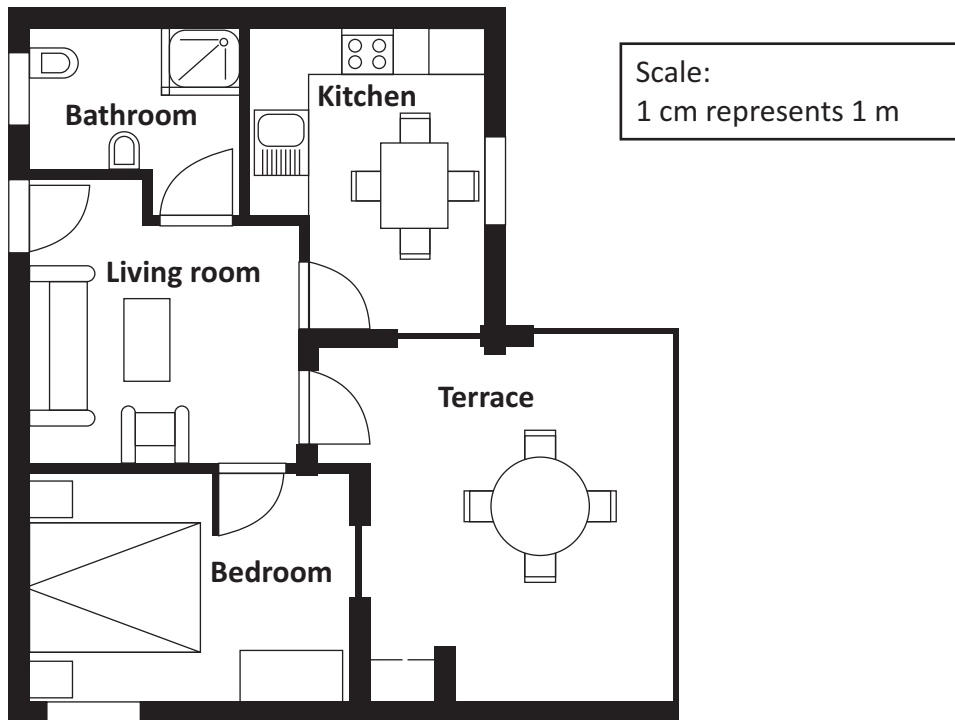
Question:-

To estimate the total floor area of the apartment (including the terrace and the walls), you can measure the size of each room, calculate the area of each one and add all the areas together. However, there is a more efficient method to estimate the total floor area where you only need to measure 4 lengths. Mark on the plan above the four lengths that are needed to estimate the total floor area of the apartment.

Explanation:-

This question is based on **Space and Shape (content area)** with **Personal Context** and checks whether a student is able to **formulate.** The student has to use spatial reasoning to show on a plan (or by some other method) the minimum number of side lengths needed to determine floor area.

1. Employing mathematical concepts, facts, procedures, and reasoning :

This involves using mathematical concepts, procedures, facts and tools to derive a mathematical solution. The **employing process** indicates **how well students are able to perform computations and manipulations and apply the concepts and facts that they know** to arrive at a mathematical solution to a problem formulated mathematically. Let us take a PISA sample question to understand this process.

Example:

Infusions (or intravenous drips) are used to deliver fluids and drugs to patients. (7)

Nurses need to calculate the drip rate, **D**, in drops per minute for infusions. They use the formula **D = dv /(60n)**

where **d** is the drop factor measured in drops per millilitre (ml)

v is the volume in ml of the infusion

n is the number of hours the infusion is required to run.

Question:- A nurse wants to double the time an infusion runs for. Describe precisely how D changes if n is doubled but d and v do not change.

Explanation: The question is meant to see the effect that doubling one variable in a formula has on the resulting value if other variables are held constant. The mathematical content area assessed is **Change and relationships** and the Context is **Occupational** and the process tested is whether a student is able to **employ** a formula.
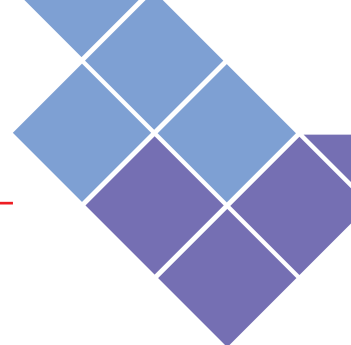
2. Interpreting and evaluating the mathematical outcomes :

This process involves reflecting upon mathematical solutions or results and interpreting or evaluating them back in the context of the initial problem. The interpreting process indicates how effectively students are able to reflect upon mathematical solutions or conclusions, interpret them in the context of a real-world problem, and determine whether the results or conclusions are reasonable. Let us take a PISA Sample question to understand this[7].

Example:

A pen drive is a small, portable computer storage device. Smita has a pen drive that stores music and photos. The pen drive has a capacity of 1 GB (approx 1000 MB). The graph below shows the current disk status of her pen drive

Pendrive disc status

- ■ Music (650 MB)
- □ Photos (198 MB)
- ■ Free Space (152 MB)
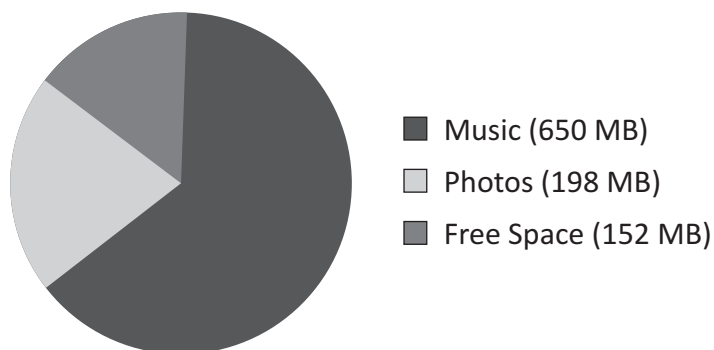
Smita wants to transfer a photo album of 350 MB onto her pen drive, but there is not enough free space on the pen drive. While she does not want to delete any existing photos, she is happy to delete up to two music albums. Smita's pen drive has the following size music albums stored on it.

Album 1	100 MB
Album 2	75 MB
Album 3	80 MB
Album 4	55 MB
Album 5	60 MB
Album 6	80 MB
Album 7	75 MB
Album 8	125 MB

By deleting at most two music albums is it possible for Smita to have enough space on her memory stick to add the photo album? Circle "Yes" or "No" and show calculations to support your answer.

Answer: Yes / No

....

....

....

Explanation: Here the student has to compare and calculate values to satisfy given criteria. The Mathematical content area used is **Quantity** under **Personal** context and the Process is" **Interpret"**.

> **Note to teacher:** It may be kept in mind that PISA has no specific syllabus; rather it is intended to test the Mathematical Literacy in the content area of Quantity, Space & Shape, Change & Relationships and Uncertainty & Data. It is important to do content analysis and identify the units where the related concepts are learnt. Questions may be prepared for each unit, which will test whether the students are able to employ mathematical concepts, facts, procedures and reasoning and are skilled in interpreting, applying and evaluating by connecting them to day to life. These questions may be used for enhancing the learning.

4. EFFECTIVE TRANSACTIONAL STRATEGIES FOR ENHANCING MATHEMATICAL LITERACY

Imparting Mathematical Literacy is crucial for helping children to lead a successful life in the 21st century. Mathematisation is an important process that every teacher should internalize for effectively transacting the Mathematics curriculum. Let us understand this process in detail.

4.1 Process of Mathematisation

For making use of Mathematics in daily life, children should know about mathematising the situation that they encounter. Mathematisation is a process of thinking about and making sense of the world with a mathematical lens. The National Curriculum Framework 2005, which sought to introduce new mathematical practices into classrooms considered " Mathematisation of the child's thought processes " as one of the main goals of Mathematics education in schools. Mathematics is required to live effectively in the 21st Century. Thus children should learn how to mathematise and use Mathematics to solve real-life problems of the 21st Century. Failure to use mathematical notions lead to poorly informed decision making in one's personal and professional life.

Let us understand the concept of Mathematisation with one example. (5)

> **Problem:**
> A municipality has decided to construct a streetlight in a small triangular park so that it illuminates the whole park. Where should it be placed?

This problem can be solved by mathematising the situation. The five aspects of Mathematisation are :

1. **Identify the problem in the context**: The problem is to find out location of the streetlight which can illuminate the whole Park.

2. **Organizing the problem with the help of Mathematical concepts:**

   The Park can be represented as a triangle and light can be considered as a circle with the centre at the location of the streetlight.

3. **Converting the real problem into a Mathematical Problem:** Locating the centre of the circle that circumscribes the triangle.

4. **Solving the Mathematical Problem:** The circle with the centre as the point of intersection of the perpendicular bisector of two sides will circumscribe the triangle.

5. **Making sense of Mathematical solutions in real life:** Reflecting on the solution and understanding the solution in the context is important. Suppose if one of the angles of the triangular park is obtuse, then the streetlight may be outside the park or what should be done if there are so many plants/trees which obstruct the light etc.

The above process can be represented diagrammatically as under ,where 1, 2, 3, 4 and 5 represents the steps described above :

Real solution	←5—	Mathematical solution
Real-world problem	—1,2,3→	Mathematical problem

Real World | Mathematical World
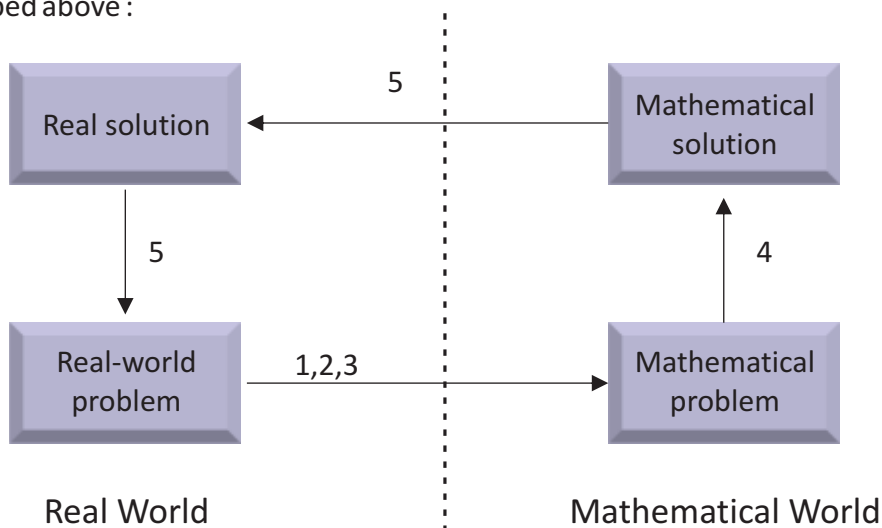
Students find it very difficult to convert the real-world problem into a mathematical problem and contextualize the mathematical solutions. Conscious efforts must be taken by the teachers to internalize this process. However any such effort should not dilute the mathematical problem solving abilities of our students. Let us try to understand the processes and strategies for effective classroom transactions promoting Mathematical Literacy by helping children to do mathematisation.

4.2   Communicating in the Language of Mathematics

Mathematics is a language similar to any other language. Understanding the vocabulary of Mathematics is very important for mathematisation. In our daily life we encounter a language like: ¼ kg apple costs Rs.100. How many grams of apple will I get for Rs 150? This question can be expressed in mathematical language as:

If 250.x= 100 and y.x=150   , Find y.

Expressing real-life situations into mathematics is possible **if we can Identify a pattern** by looking into connection, similarity, structure, rules etc. Following are the examples of some patterns which we find in day to day life.

A Mathematics teacher should help the learner to identify the pattern in different day to day life situations.
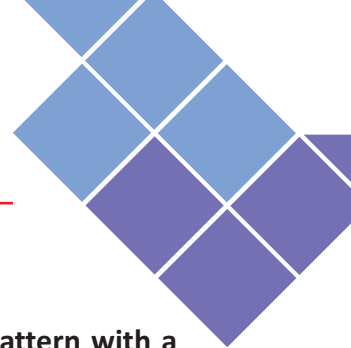
The second stage in expressing a situation mathematically **is representing this pattern with a language**. Hence the teacher should make the students understand different notations, symbols, rules etc used for expression in Mathematical language by giving suitable examples. It is also important that the students realize that the expressions in Mathematical language may have multiple meanings as per the context . For example: **4 by 3** can have following meanings.

If we use base as ten then 4/3 number can be expressed:

$$4/3 = 1.3333333…$$

Changing the base to 2, we can express **4 by 3** as 1.0101010… and if the base is changed to four we can express it as 1.1111111…… .

It is not necessary that we express **4 by 3** with numbers. It can also be expressed pictorially as given below:
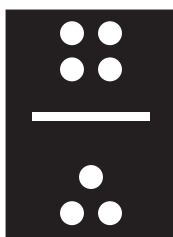
We can also express **4 by 3** by a rectangle with a diagonal equals to 5

$$\frac{4}{3}$$
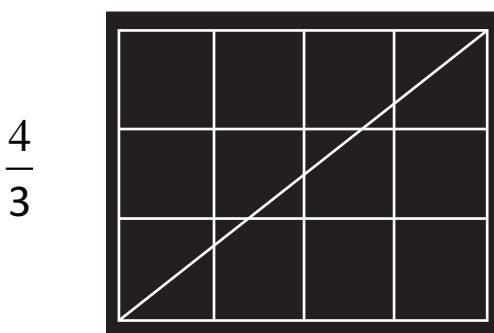
Further it can be shown as a ratio of lengths as most of the standard mobile phones or computer screens are in this ratio.

$$\frac{4}{3}$$

800x600
1024x768
1600x1200
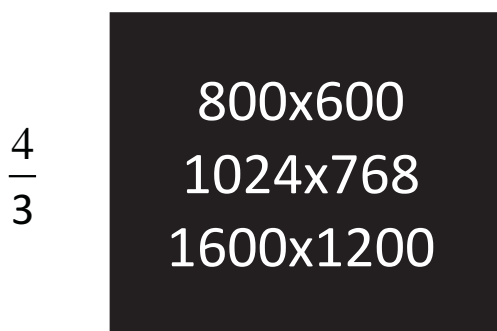
It is clear from the above discussion that rules of Mathematical language with its limitations should be communicated to students so that they will be able to express in Mathematical language.

Case Study : Example of expression in Mathematical language

A Capacity buildng programme of Facilitators of Mathematical Literacy was organized by the CBSE at DAV Model School, Chandigarh during 23rd to 24th July 2019. The following Challenge was posed to the Participants of this Capacity Building Programme immediately after the receipt of the group photograph .

Challenge: Identify your location in the group photo and express it in mathematical language

The teachers expressed their position in mathematical language using different vocabulary. This shows the beauty of mathematical language.

1.   If Pramod sir is considered as origin then my coordinates are (-2,2)

-Pardeep Singh

2.   If distance between two persons is 1unit...and Taking Pramod sir at one of the vertices of the right-angled triangle...then I am at the distance of $\sqrt{26}$ units from Pramod sir.

-Gurpreet Kaur

3.   Find me on the intersection of the line at the back of the seated ones and the line joining the midpoints of the first and the last line. You move a step forward from me you get blues, a step backward you will get checks.

-Pooja Sharma

4. I am a pink rose, my position is hidden in variable of equation $4x$-16=0 (any row). Guess who am I?

    -Sangeeta Gill

5. I am at right most corner of a quadrilateral formed by first 4 persons from left side.

    - Gurleen Kaur

6. We are in an elliptical shape. Pramod sir is in its centre , my position is (r cos150, r sin150)

    -Dr. Rajendra Vadhwani

7. In the second number line if Ma'am in orange suit is taken as the origin then I am at an even position. My highest factor is 6.

    -Simranjeet Kaur

8. Jokes apart..I feel I am one side of trapezium and the other parallel side is the honorable guest sitting on left of Mr. Pramod and two persons behind him.

    -Gurpreet kaur

9. In the second row, whether count my position from the right or from the left, the one's place digit remains the same composite number. You'll reach me late if you count from left.

    - Poonam Kukreja

10. If I consider sir sitting left to Pramod sir as origin then I am on the intersecting point of line represented by y=x-1 and y=1,considering the distance between two adjacent persons to be 1 unit.

    Sir if I consider you as origin then I am on the point where the slope is zero for the curve y-1=(x-1)^2

    -Shifali Singla

11. I am the inner vertex of triangle formed by males on left side.

    -Navjot Singh

12. Wow! What a Mathematical position I am standing at! I am the Orthocentre, Circumcentre, Incentre and centroid of an equilateral triangle whose two vertices are Promod sir and Anuja ma'am.

    -Shelly Rajput

13. If Pramod Sir is considered as origin and origin is shifted at the point (10,1), I will be just behind him at the point (0,1) in the new coordinate system.My full face will also be visible as he will be sitting on the chair.

    -S.N.Tripathi

14. I have occupied the 3/4th space of extreme left cylindrical shaped pillar. Other than coordinate geometry.

    -Pooja Puri

4.3   Changing the focus of teaching:

The ability to mathmatise can be enhanced in children with conscious efforts by the Mathematics teacher. He/She must incorporate such problems in day to day classrooms and help the learner to mathematise real-life problems. The present focus is on: **teach some Mathematics, practice some Mathematics, apply some Mathematics** must be changed in order to achieve this goal.

To mathematise and solve the real-life problems, the children should be able to :

1)   contextualize and formulate it as a mathematical problem.

2)   do the Mathematics, and employ skills and knowledge.

3)   interpret and evaluate the outcomes of the mathematisation and reflect on how Mathematics result applies and fits in with the real world.

Mathemitsation is the most difficult task and is also the MOST important one to which we do not give much attention. Our emphasis is to teach and give some practice questions to use Mathematics tools and techniques without relating them to life. Then if we have time, we ask the student to apply Mathematics to a word problem which is not connected to their immediate life. It is important that teachers:

• **Use a problem solving, investigative, open-ended approach** — use real texts and real situations make connection between Mathematics and the real-life

• **Start from the real world**—teach students how to identify and extract the Mathematics from real-life situations that they are likely to face

• **Make the Mathematics   explicit** and then, when the need arises, or gaps in knowledge appear, teach the Mathematics that is required[9].

Integration of the above in the classroom will help the children to make use of Mathematics to the maximum.

4.4   Effective classroom strategies based on 5 Es:

The 5 E's model emphasise on the fact that learners build ideas on the basis of their previous knowledge. The 5 Es can be used with all kinds of learners including adults.

Each phase of learning is described by an **"Engage, Explore, Explain, Elaborate and Evaluate".** The 5 Es allow students and teachers to experience common activities, to use and build on prior knowledge and experience, to construct meaning, and to continually assess their understanding of a concept.
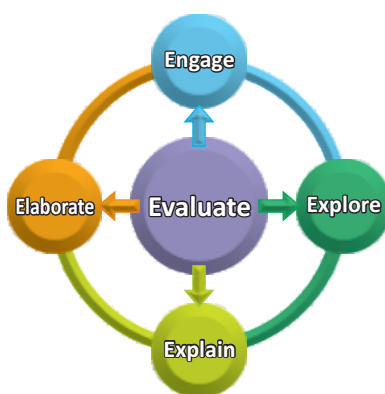
Engage: The first step is to ensure the readiness of the learner. For any meaningful learning, the learner must be ready. For creating interest in the learning , situations should be created so that he/she is able to establish connections between previous knowledge and present learning experiences. Students should be mentally engaged in the concept, process, or skill to be learned. This can be done through storytelling, presenting real-life problems related to the topic etc. Teachers must be able to connect different concepts of Mathematics to the daily life and previous knowledge of the learners.

Explore: During this stage, students actively explore their situation or manipulate materials. Teacher must plan activities so that students are able to explore and reflect on various aspects of the concepts. Sufficient time should be allocated for this phase of exploration. Appropriate level of challenge should be given to leaner to motivate them to explore during this stage. This stage can also be used for improving the reading skills of learners, if contextualized problems are provided to the learners with sufficient reading load.

Explain: Students are given the opportunity to explain after the exploration. They may be given sufficient time to express their ideas without fear. Teachers should be non-judgmental and should express empathy towards the learners. Formal terms, definitions, and explanations for concepts, processes or skills can be defined during the stage with the help of students. This stage also helps the learner to strengthen their communication skills.

Elaborate: This is the stage of practicing the skills or behavior learned. Students get opportunities to refine their skills, gain more information or change their behavior during this stage. The students may be given opportunity to find out the connections with other subjects during this stage.

Evaluate: Teachers get the opportunity to assess the students' performance to understand strength and weakness. Students will also be able to assess their performance during this stage.

How Children Learn : Inputs from National Curriuclum Framework 2005 (1)

Children learn in a variety of ways

- through experience,
- making and doing things,
- experimentation,
- reading,
- discussion,
- asking,
- listening,
- thinking and reflecting,
- expressing oneself in speech,
- expressing oneself in movement,
- expressing oneself in writing,
- both individually and with others.

- Learning **takes place both within school and outside school.** Learning is enriched if the two arenas interact with each other.
- **All children are naturally motivated to learn and are capabe of learning**
- Learning **must be paced** so that it allows learners to engage with concepts and deepen understanding, rather than remembering only to forget after examinations.
- **Learning can take place with or without mediation** The social context and interactions provide avenues for learners to work at cognitive levels above their own. At the same time learning **must provide variety and challenge, and be interesting and engaging.** Boredom is a sign that the task may have become mechanically repetitive for the child and of little cognitive value.
- Schools must provide **opportunities to question, enquire, debate, reflect, and arrive at concepts or create new ideas.** An element of challenge is critical for the process of active engagement and learning various concept, skills and positions through the process.
- Learning takes place through **interactions with the environment around, nature, things and people, both through actions and through language.** The physical activity of moving, exploring and doing things, on one's own, with one's peers or in the company of adults, and using language —to read, to express or ask, to listen and to interact —are the key processes through which learning occurs. The context in which learning takes place is thus of direct cognitive signicance.
- In the early primary school years, a beginning has been made in the area of group work. **projects and activities that can be carried out by groups need to become a feature of learning in middle and high school also.** There are ways in which such group learning can be assessed and evaluated. Schools could also consider giving mixed age groups of children projects to do together.
- **Making, meaning and developing the capacity for abstract thinking, reflection and work are the most important aspects of learning.**
- We must ask ourselves why we only ask children to g*ive a*nswers to questions. **Even the ability to make a set of questions for given answers is a valid test of learning.**
- **Art and work provide opportunities for holistic learning** that is rich in tacit and aesthetic components.
- **Intelligent guessing must be encouraged as a valid pedagogic tool.** Quite often, children have an idea arising from their everyday experiences, or because of their exposure to the media, but they are not quite ready to articulte it in ways that a teacher might appreciate.

4.5 Teaching Problem-Solving Skills:

In our course of teaching we ask students to solve "problems". Can we consider these as Problems or just exercises? In problem-solving ,we use critical thinking and decision making but exercises are meant only for consolidating the learning. Real problem solving is the one where the situation is unknown to the student. Students and teachers must be aware of the **principles of problem-solving** which are given below:

- **Teach within a specific context:** Use of real-life situations while teaching will help in contextualization. Problem-solving skills should no be used as an abstract skill in isolation to the context.

- **Help students understand the problem:** Students should be able to understand the final goal. The questions like What and Why will help the students to find out the How.

- **Take enough time:** Sufficient time should be given to the students to understand the problem, suggesting tentative solutions, reflection etc.

- **Ask questions and make suggestions:** To help the children to develop Analytical skills, questions should be asked by the teacher. Questions should also be asked to reflect on the method used and the difficulties they face.

- **Link errors to misconceptions:** Use the errors committed as misconceptions and teachers should help the children to improve.

Students should be given opportunities to approach the problems methodically and solve them. They should be able to follow the following steps while solving a problem related to life.

1. **Understand or define the problem:** Understanding or defining a problem or a task is the most important aspect of any Problem-solving approach. During this process, one has to understand the context, concepts involved, the unknown parameters and constraints. Students should be asked to list out the known elements and the unknown parameters. They may be given opportunities to identify the constraints which are either stated or implied. Students may be asked to reflect upon the nature of results and whether that is expected or not.

2. **Analyzing the Problem:** Use this stage to ponder the problem. Ideally, students will develop a mental image of the problem at hand during this stage. Students may be helped to find out different aspects of the problem and may be given a chance to connect the same with their own knowledge:

3. **Identify different possible solutions:** Different problems will need different problem-solving approaches. In general they are computed by : simplify, use an equation, make a model, diagram, table, or chart or work backwards. There can be multiple solutions for any problem. One of the most obvious solutions may not be the best one.

4. **Choose the best solution:** Based on the pros and cons of each proposed solution, the students must be able to find out the best solutions. While selecting the best solution feasibility, effectiveness, time etc may be taken into consideration.

5. **Implement the Solution:** It should be told to the students, that most problems are not solved quickly or on the first attempt. While carrying out the plan one should also take into consideration the constraints.

6. **Evaluate the outcome:** students should be encouraged to reflect. Once a solution has been reached, students should ask themselves whether the results are relevant to the situation.

You may see how above steps can be used in solving the problem given below:

Form the Box with Maximum Volume

Context: Here is a 20cm x 20 cm square. Gurmeet wanted to make a box by cutting the squares of integral length from the corner and by folding the flaps along the sides.
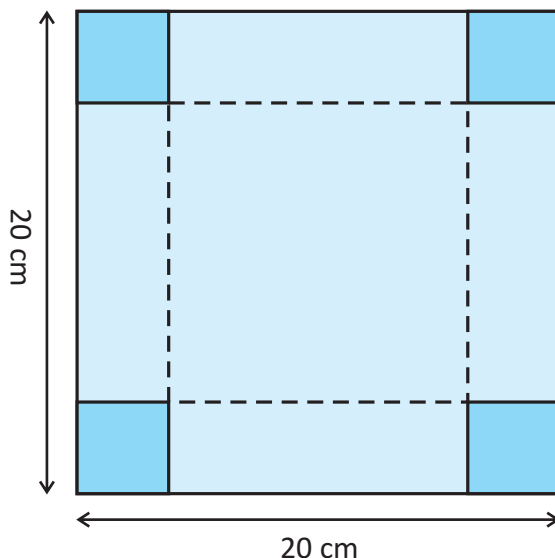
Following questions will help in going through different processes of problem- solving approach :

- What are the possible different sizes of boxes Gurmeet can make? List all such boxes.
- Which box will have maximum volume?
- Which box will have minimum volume?
- Is there any relation between size of the square cut out and volume of the box? If yes, can you put it in the form of an equation?
- What will happen if sides of the square are not integral length?

The students should be allowed to ask questions and also be given time to reflect on these questions for effective problem-solving.

IMPORTANT POINTS WHILE DESIGNING LEARNING EXPERIENCES :

(Reference : National Curriculum Framework 2005 (1))

- Tasks that are too easy or too difficult, that are repetitive and mechanical, that are based on recalling the text, that do not permi self-expression and questioning by the child and that depend solely on the teacher for correction, make the child assume the passive stance of obedience. Learners learn not to value their own ability to think and reason, that knowledge is created by others and that they must only receive it.
- Instead of a culture of quizzing, of answering quickly and always knowing the right answer, we need to allow learners to spend time on deeper, meaningful learning.

An example of Self-learning Material based on effective transactional strategies.

INTRODUCTION TO POLYNOMIAL

Objectives:

This unit will help the learner to :

- express some life situations in mathematical language using polynomials

- appreciate the importance of polynomials in day to day life

- express the key terminologies of a polynomial

- solve mathematical problems using polynomials

Concept Map:

```
                    ┌─────────────┐
                    │ Polynomial  │
                    └─────────────┘
          ┌───────────────┼───────────────┐
┌─────────────────┐ ┌─────────────────┐ ┌─────────────────┐
│                 │ │   Types of      │ │                 │
│                 │ │ polynomials and │ │  Application    │
│ Characteristics │ │   key terms     │ │  in daily life  │
└─────────────────┘ └─────────────────┘ └─────────────────┘
```
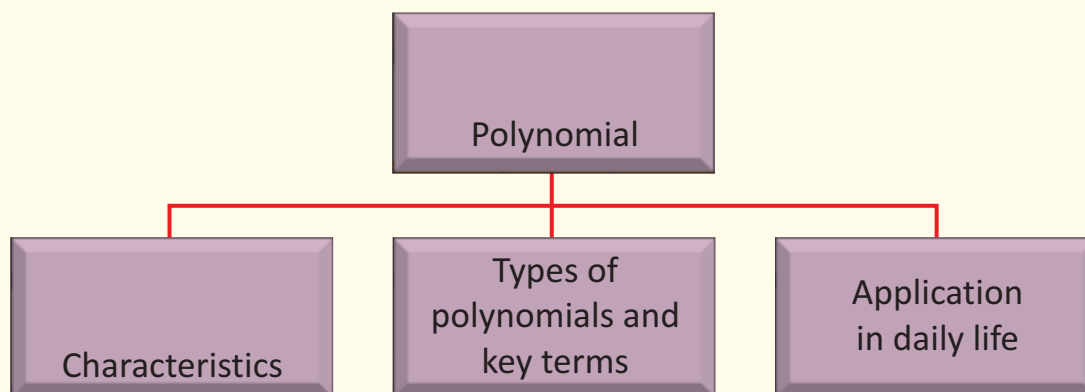
The word 'Polynomial' is derived from two words poly and nomial : poly- (meaning "many") and nomial (in this case meaning "term").   You may wonder how learning polynomials will be helpful in our lives. See what Seema wants to achieve and how can you help her using the concept of polynomials

Seema is an onlnetaxi driver and she has set a target to earn **Rs. 1000** on monday. The meter charges the customer at a **rate of Rs 10 per kilometer** for travelling. But the driver **gets only half of the amount paid** by the traveler after deduction of the commission and other charges by the taxi company.

Challenge: How many kilometres should she drive to achieve her target on Monday?

Solution :

If x is the distance covered by her in km, she will earn an amount equal to the product of rate of earning per km (10/2) and distance in km (x) .

$$\text{i.e } (10/2)x = 5x$$

If she wants to earn Rs. 1000, then 5x must be equated to 1000

$$\text{i.e } 5x = 1000$$

Therefore value of x = 200 km . Hence, she should drive 200 km on Monday to earn Rs. 1000.

PS : See how her target will change if she wishes to earn less or more (say Rs. 500 or Rs. 2000) money :

- If she wants to get Rs. 500 then she should equate 5x = 500 and calculate her target as 100 Km

- If she wants to get Rs. 2000 then she should equate 5x = 2000 and calculate the value of x.

Here we have expressed a real-life problem in mathematical language to solve it. In this case **5x is called a polynomial in the variable x.** Some other examples of polynomials are :

$$x^2, \quad 2x+3, \quad 2x^2+3x+2.$$

What are the components of a Polynomial ?
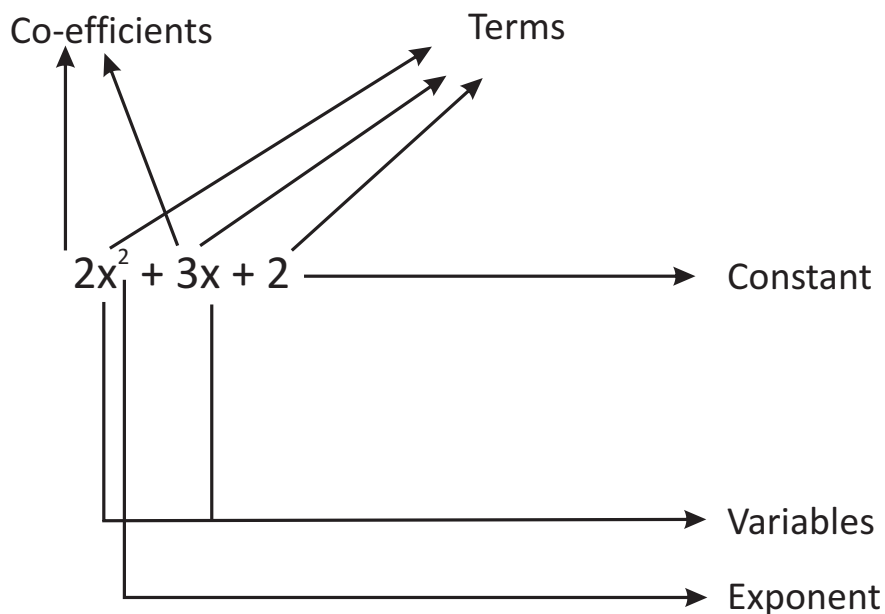
$2x^2$, 3x and 2 are parts of the polynomial and are called **terms** of a polynomial . Thus a **poly_nomial** has **many_terms.**

The power of the variable in a polynomial is called the **exponent**

The number multiplied by the variable is called a **coefficient**

How do you know that an algebraic expression is a polynomial ?

Recall your understanding of an algebraic expression. You can easily see that **polynomials are also algebraic expressions**. Then what are the characteristics of a polynomial ?

Challenge : Find out the characteristics of polynomials from the following table and see if you can define a polynomial.

Polynomial	Algebraic expressions which are not polynomial
2x	$2x^{-1/3}$
3x	$3x^{-1}$
z^2+1	$1/z$
$X^3 + X^2 + 3$	$z^{-1}+1$
2	$y^{-3} + y$
p+2	$\sqrt{3x}$

What is a Polynomial?

Algebraic expressions with whole numbers as exponents of variables are termed as polynomials.

That means : if the exponents are -1, 2 , 5 , 1/3 etc (any fraction or negative number) then such an algebaic expression cannot be considered as a polynomal. In short they are the simplest forms of algebraic expressions where exponents are only whole numbers.

Challenge: Give new examples of the following

S. No.	Definition and example	Write another example
1.	Polynomials having only one term are called **monomials** Example : 2y	
2.	Polynomials having only two terms are called binomials Example : 2x+1	
3.	A number is a constant polynomial Example : 3 ,-1	
4.	The highest power of the variable in a polynomial is called the degree of the polynomial. Example : degree of $2X^3 + X^2 + 3$ is 3 degree of $X^2 + 3$ as a polynomial is 2	

Challenge:

Write two different polynomials that describe the area of the garden shown on the right.
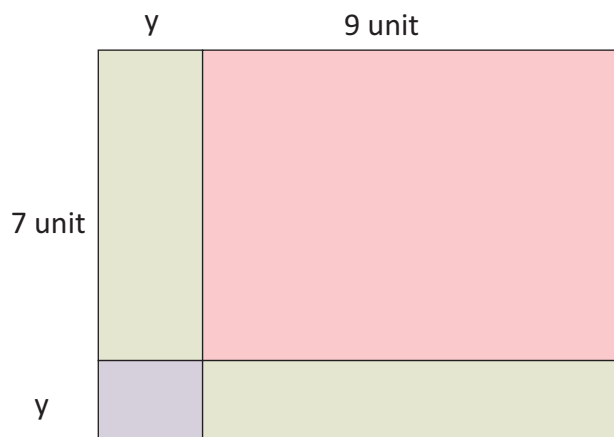
Solution :

First Method:

First, we will define the polynomial that describes the area of the rectangle as a single figure.

We are tasked with writing an expressions for the area of the figure above. The area of a rectangle is given as A=lb.

We need to consider the whole figure in our dimensions. Let us write the polynomials :

$$l=(y+7) \quad \text{and} \quad b=(y+9)$$

We can multiply these two polynomials to get the area as under :

$$\text{Area}= (y+7)(y+9) = y^2+16y+63$$
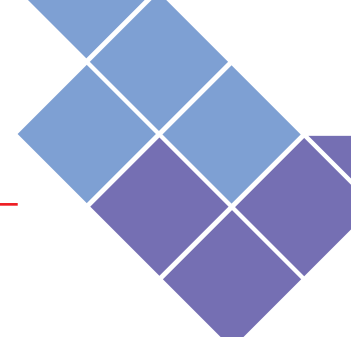
Second Method:

Now we will find an expression for the area of the whole figure as comprised by the areas of the four rectangles added together.
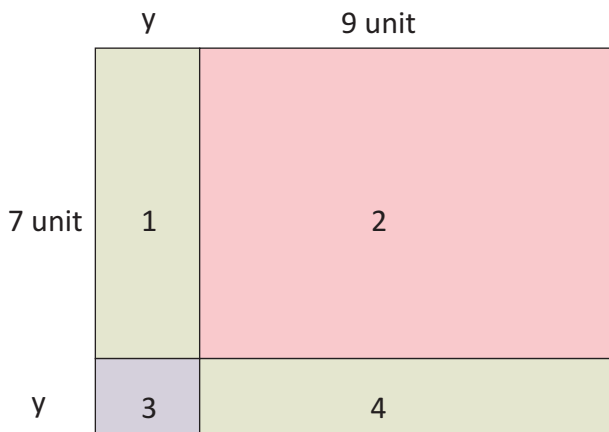
Let us write down the polynomials corresponding to the area of each rectangle . Area of each part can be expressed as under:

$$A_1 = 7y$$

$$A_2 = 7 \times 9 = 63$$

$$A_3 = y \, xy = y^2$$

$$A_4 = y \times 9 = 9y$$

We have four polynomials $7y, 63, y^2, 9y$

In the above examples we have written the polynomials : $7y$, 63, y^2 , $9y$, $y^2 + 16y + 63$

General form of a Polynomial with 'n' degree :

We can write a general form a of polynomial with n degree as under:

$$a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots a_1 x + a_0$$

where a_n is the coefficient of $(n+1)^{th}$ term, a_{n-1} is the coefficient of nth term and so on.

Did you know ?

(source: www.courses.lumenlearning.com)
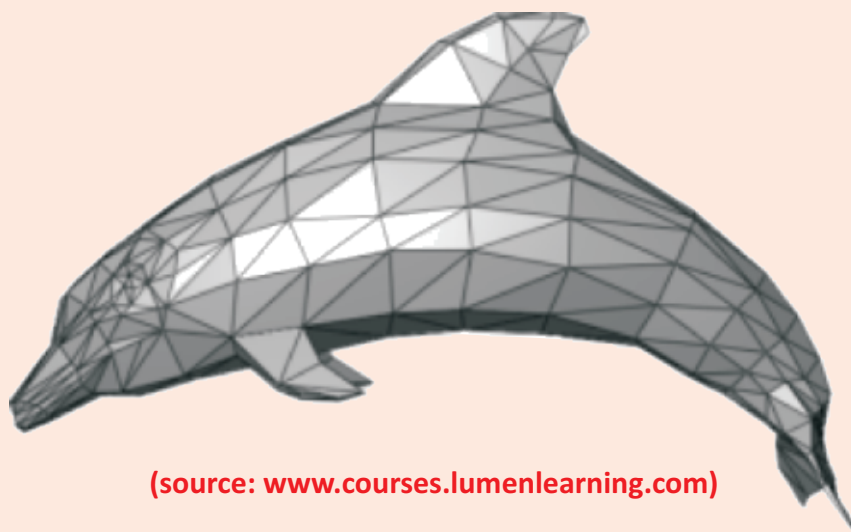
The dolphin in the image was created by plotting points in space to create connected triangles. This method of rendering graphics works well and is in wide use, but it takes a lot of computer memory. Recently, researchers have been investigating the use of polynomials for rendering graphics in part because it demands less memory in the process. In this process, the surfaces that are rendered are made from solutions to algebraic polynomials.
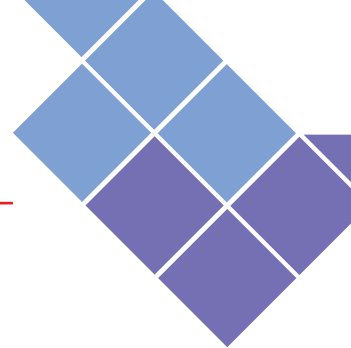
Challenge : Find out another concrete example of a polynomial used in day to day life. Also write down the degree and type of the polynomial.

Or

Find out real life application of the polynomials : x^2, $2x+3$, $2x^2+3y+2$.

Conclusion :

You need to know that Polynomials are an important part of the "language" of Mathematics and Algebra. They are used in nearly every field of Mathematics to express numbers as a result of mathematical operations. Polynomials are also "building blocks" in other types of mathematical expressions, such as rational expressions.

Homework:

Find out an example of a multivariable polynomial , which you use in day to day life.

One Solution

You've probably used a polynomial in your head more than once when shopping. For example, you might want to know how much three kg of flour, two dozen eggs and three packets of milk cost. Before you check the prices, construct a simple polynomial, letting "f" denote the price of flour, "e" denote the price of a dozen eggs and "m" the price of a packet of milk. It looks like this: 3f + 2e + 3m.

This basic algebraic expression is now ready for you to input prices.

If flour costs Rs. 50, eggs cost Rs 20 a dozen and milk costs Rs. 20 a packet, you will be charged 3x50+2x20+3x20.

Note to teacher; Now ask students to Solve IX NCERT exercise 2.1. Please reflect on whether contextualization has reduced Mathematical rigour which is there in the NCERT text book.

4.6   Emphasizing the daily life connection of Mathematics:

In addition to the classroom strategies in which the mathematical concepts are derived from the examples in day to day life; opportunities should be given to students to apply the same in real life. Parents and teachers can play a vital role in this activity. Teachers should be able to identify the situations in day to day life and pose questions to the students. Home assignments given to the students can be connected to their daily life. Parents should consciously take care of the application of Mathematics in daily life of their ward. The following example gives a situation where a student visits a shop and tries to make use of his learning of Mathematics.

Case study:

Scenario : 1

Arif went to a shop to purchase shoes. As he was not aware of the size of his shoe, he was asked to measure the size by the shopkeeper using the measuring scale shown below **.**
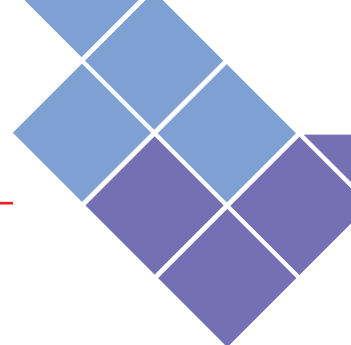
He measured his shoe size and found it to be 7. While searching for a suitable shoe, he also finds some tags like the one given below attached to a girl's sandle.

wgfl w		
205734-6PU		
US	CAN	ES
w8	f8	m8
EUR	UK	JP
38-39	6	24
CN	KR	BR
250(2)	250mm	37

1   9 1 4 4 8   3 0 8 0 4   6

Being curious, he wanted to know more about the shoe sizes and searched for the information on internet and found the following information:

There are a number of different shoe-size systems used worldwide. While all of them use a number to indicate the length of the shoe, they differ in exactly what they measure, what unit of measurement they use, and where the size 0 (or 1) is positioned. Some systems also indicate the shoe width, sometimes also as a number, but in many cases by one or more letters. Some regions use different shoe-size systems for different types of shoes (e.g. men's, women's, children's, sport, and safety shoes). In practice, shoes should be tried on for size and fit before they are purchased. (Source :en.wikipedia.org/wiki/shoe_size)

On further searching he finds out the conversion details from a company (source: www.myshoes.ng) as given below:

Ladies Shoe Sizes													
British	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7	7.5	8
European	34	35	35.5	36	37	37.5	38	38.5	39	39.5	40	41	42
American	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5
Japanese (cm)	21.5	22	22.5	23	23	23.5	24	24	24.5	25	25.5	26	26.5

Mens Shoe Sizes													
British	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5	12
European	38	38.7	39.3	40	40.5	41	42	42.5	43	44	44.5	45	46	46.5	47
American	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10	10.5	11	11.5	12	12.5
Japanese (cm)	23.5	24	24.5	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5	30	30.5

Girls Shoe Sizes													
British	8	8.5	9	9.5	10	10.5	11	11.5	12	12.5	13	13.5	1	1.5	2	2.5
European	26	26.5	27	27.5	28	28.5	29	30	30.5	31	31.5	32.2	33	33.5	34	35
American	8.5	9	9.5	10	10.5	11	11.5	12	13.5	13	13.5	1	1.5	2	2.5	3
Japanese (cm)	14.5	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5	22

Boys Shoe Sizes													
British	11	11.5	12	12.5	13	13.5	1	1.5	2	2.5	3	3.5	4	4.5
European	29	29.7	30.5	31	31.5	33	33.5	34	34.7	35	35.5	36	37	37.5
American	11.5	12	12.5	13	13.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Japanese (cm)	16.5	17	17.5	18	18.5	19	19.5	20	20.5	21	21.5	22	22.5	23

Question:

1. Refer to the details given in the tag attached to a girl's sandal given earlier. Check and confirm whether the information given in the table above matches with that given on the tag. Point out the differences with justification.

2. Also confirm whether there is a fixed ratio between the different numbers given in the table. Justify your answer.

Scenario : 2

On the purchase of shoes, a gift voucher worth Rs 300/- shown below was given to Arif with a condition that he cannot purchase an item with the voucher for an amount less than the amount printed on it

Question: If a pair of socks costs Rs 100 and the cost of a small purse is Rs 55, what is the maximum number of articles that can be purchased by Arif by spending a minimum additional amount above the value of the gift voucher?

Challenge to teachers: Frame questions based on the bill that Arif received after the purchase of the shoes. You may try to connect the information to learning outcomes at different levels/classes. The bill can also be compared with a similar bill shown below when VAT was in place of GST.

```
┌──────────────────────────────────────┐
│                                        │
│            Sunil Agency                │
│         Shop No. 02 Zirakpur           │
│           Punjab - 140603              │
│                                        │
│  ------------------------------------  │
│                  VAT Bill              │
│  ------------------------------------  │
│   Cash      CS16/11027     28/08/2016  │
│                                        │
│  ------------------------------------  │
│   Item Dtls  Qty.      Rate     Value  │
│  ------------------------------------  │
│  091202309633/  1.00  4495.00  4495.00 │
│  Less Discount (#111)   50%     2247.50│
│                 =====          ========:│
│  Sale Total      1.00           2247.50│
│                                    .50 │
│                 =====          ========:│
│  NET BILL AMOUNT                2248.00 │
│  Tender                                │
│  Cre      Master C.S. No. 2117  2248.00│
│  Thank U, Visit Again                  │
│  Have a Nice Day                       │
│           5:10:21  PM                  │
│                                        │
└──────────────────────────────────────┘
```

Note to teacher: Classroom transactions should focus on mathematisation. Students should be able to convert a real life problem into a Mathematical problem and the solution provided be reflected upon for its relevance. It is also important that new concepts may be introduced connecting the same to daily life and students must be convinced about the utility. Above all the problem-solving skill should be developed among students.

5. CROSS-CURRICULAR LINKAGES

No subject can exist in isolation. To understand Mathematics, one should have the knowledge and the skills of language, Science and the Social Sciences. Moreover, Mathematics competencies are required to develop competencies in other subjects. The process of Mathematisation necessitate this linkage.

5.1 Exploring other subjects for cross curricular linkages

Mathematics is used in all subjects for expression. It is important that we identify the components of Mathematics in other subjects. Let us refer to a question given in one of the units of Economics, IX Social Science Chapter-1: (Story of Village Palampur). NCERT text book, 2019.

The following Table1.1 shows the land under cultivation in India in units of a million hectares. Plot this on the graph provided. What does the graph show? Discuss in class.

Table 1.1: Cultivated area over the years

Year	Cultivated area (in Million Hectares)
1950-51	129
1990-91	157
2000-01	156
2010-11 (P)	156
2011-12 (P)	156
2012-13 (P)	155
2013-14 (P)	156
2014-15 (P)	155

(P) - Provisional Data
Source: Pocket Book of Agriculture Statistics 2017, Directorate of Economics and Statistics, Department of Agriculture, Cooperation and Farmers Welfare.

From the above example, it is clear that Mathematics is vital in understanding Social Science. A Social Science teacher therefore should give children the opportunity to plot the graph rather than referring to the graph given in the textbook. The graph plotted by the students can be discussed in the classroom. Alternatively, the teacher can also use the graph available in the text book and students may be asked to interpret e.g. In the same unit we can also find the following graph.

Graph 1.1: Distribution of Cultivated Area and Farmers
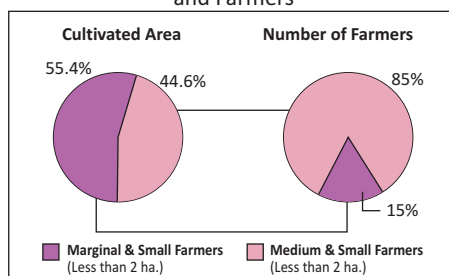
Source: Pocket Book of Agriculture Statistics 2017 and State of Indian Agriculture 2017. Department of Agriculture, Cooperation and Farmers Welfare (As per Agriculture Census 2010-11).

Let us take another example. From Chapter 3:Synthetic Fibres and Plastic of Class VIII Science in NCERT text book, Activity 3.1 given is as under:

Take an iron stand with a clamp. Take a cotton thread of about 60 cm length. Tie it to the clamp so that it hangs freely from it as shown in Fig. 3.5.
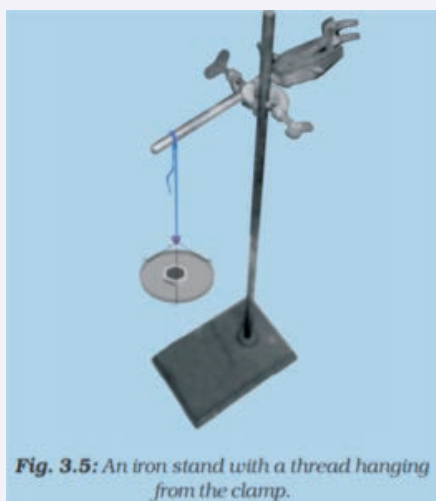
Fig. 3.5: *An iron stand with a thread hanging from the clamp.*

At the free end, suspend a pan so that weights can be placed in it. Add weights one by one till the thread breaks. Note down the total weight required to break the thread. This weight indicates the strength of the fiber. Repeat the same activity with threads of wool, polyester, silk and nylon. Tabulate the data as shown in Table

3.2. Arrange the threads in order of their increasing strength.

Observation Table 3.2

S. No.	Type of Thread/fiber	Total Weight required to break the thread
1.	Cotton	
2.	Wool	
3.	Silk	
4.	Nylon	

(Precaution: Note that all threads should be of the same length and almost of the same thickness.)

You may use a hook or a nail on the wall for hanging the fibers and a polythene bag at the other end. In place of weights you may use marbles (or pebbles) of similar size.

During this experimentation, students can be motivated to make use of various mathematical concepts. The concept of weight is related to quantity. Data obtained can be analyzed using Mathematics they have learned. The students can be asked to find out the relationship between total weight required to break the thread and thickness of the thread using the same fibre. They may internalize the concept of proportion /ratio using this activity while learning Scientific method.

It is also possible to integrate Art and other co-curricular activities with Mathematics. The beauty of Mathematics in life needs to be appreciated by the students. You may read the Art Integration document published by the Board under the Link http://cbseacademic.nic.in/web_material/Circulars/2019/art_integration.pdf to get inputs on Art integration with Mathematics.

From the above examples it is also clear that language plays a crucial role in understanding the concepts , be it Mathematics, Social Science or Science. Reading competencies are integral to problem-solving. Efforts should be made by the Mathematics teacher to discuss such topics with other subject teachers to sensitize them about the importance of mathematical concepts and how they can teach the same in their classroom. It is also important that efforts are made by Mathematics teachers to link the subject with co scholastic activities including Arts.

5.2 Concept Mapping :

For improving the learning, we need to help the learner to understand the linkages between different concepts in addition to cross-curricular linkage. Concept mapping is a type of structured conceptualization which can be used to develop a conceptual framework that can guide evaluation or planning. (Trochim, 2003). It is useful in developing an understanding of a body of knowledge. They can also be used to

➢ Explore new information and relationships

➢ Access prior knowledge

➢ Gather new knowledge and information

➢ Share knowledge and information generated

➢ Design structures or processes

➢ Writing, web pages, presentations, constructions

An example of concept map in Mathematics is given below:
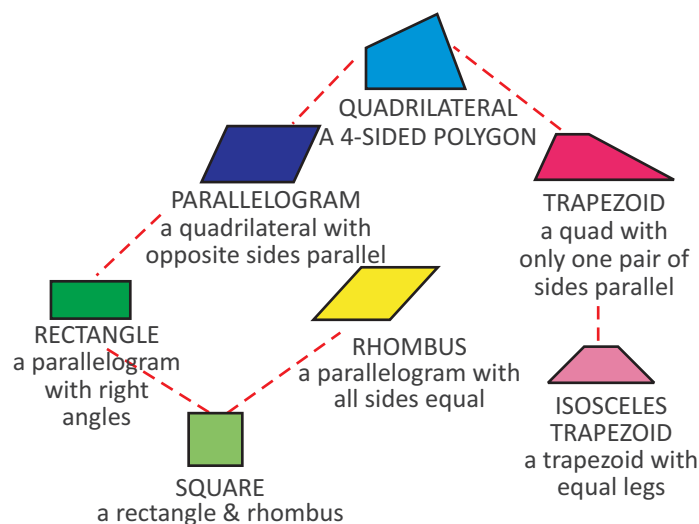
The concept map can also be used for various other purposes than using in classrooms

- ➢ Handy way to take notes during lecture.
- ➢ Excellent aids to group brainstorming.
- ➢ Planning the studies and career.
- ➢ Providing graphics for the presentations and term papers
- ➢ A way to outline the term papers and presentations.
- ➢ Refine your creative and critical thinking.

5.3 Content Analysis:

For cross-curricular linkages and concept mapping, the content analysis is very important. Most of us use text books and syllabus for our teaching. It is important that one analyzes the content to identify the concepts and appreciate the linkage between concepts. For seeing that PISA 2021 framework is not beyond our existing curriculum, we need to do the content analysis. This will help us in understanding how much the content areas deviate from the PISA Mathematical Literacy framework. An example of content Analysis mapped with the content areas of PISA Mathematical framework is given below:

Content Analysis

Class : VIII   **Mathematics**

Chapter No.	Chapter Name	Topic and the concepts	Learning outcomes	Appropriate Content Area according to PISA framework
15	Introduction to Graphs	Mean, Median, Mode of Grouped Data	The learner draws and interprets bar charts and pie charts.	Uncertainty & Data

The content analysis helps the teacher to plan the lesson by keeping in mind the previous knowledge of the students. The same exercise can also be used to identify the content area from subjects like Science , Social Science, Language etc to find out the cross-curricular linkage. This is a useful tool to get insights into the linkage between concepts and same can be used while assessing different concepts. It will be easy to make concept maps once the linkages between different concepts are identified. However while mapping the curriculum with the PISA content areas, one should also keep in mind that the concepts do not exist in isolation.
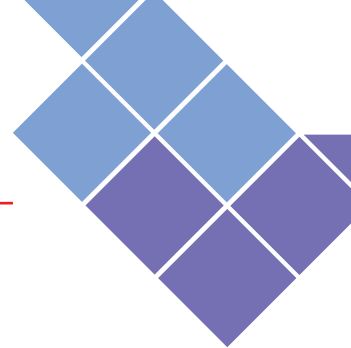
SMAPLE MAPPING OF CONTENT
(Source : Work done by
PISA Mathematical Literacy group SCERT Chandigrah)

CLASS VI

NCERT CHAPTERS	PISA LITERACY AREA	LEARNING OUTCOMES
• Ch-4 Basic Geometric Ideas	**Shape & Space**	The learner describes geometrical ideas like line, line segment, open and closed figures, angle, triangle, quadrilateral, circle, etc. with the help of examples in surroundings.
• Ch-5 Understanding Elementary Shapes	**Shape & Space**	The learner classifies triangles into different groups/types on the basis of their angles and sides e.g. scalene, isosceles or equilateral on the basis of sides, etc. The learner classifies quadrilaterals into different groups/types on the basis of their sides/angles. The learner identifies various (3-D) objects like sphere, cube, cuboid, cylinder, cone from the surroundings. The learner describes and provides examples of edges, vertices and faces of 3-D objects
• Ch-10 Mensuration	**Shape & Space**	The learner finds out the perimeter and area of rectangular objects in the surroundings such as floor of the class room, surfaces of a chalk box etc.
• Ch-13 Symmetry	**Shape & Space**	The learner demonstrates an understanding of line symmetry by —identifying symmetrical 2-Dimensional (2-D) shapes which are symmetrical along one or more lines. —creating symmetrical 2-D shapes, reflections too.

• Ch-14 Practical Geometry	**Shape & Space**	The learner demonstrates an understanding of angles by ––identifying examples of angles in the surroundings. —classifying angles according to their measure —estimating the measure of angles using 45°, 90°, and 180° as reference angles
• Ch-11 Algebra	**Change & Relationship**	The learner uses variables with different operations to generalise a given situation e.g. Perimeter of a rectangle with sides x units and 3 units is $2(x+3)$ units
• Ch-12 Ratio & Proportion	**Change & Relationship**	The learner compares quantities using ratios in different situations e.g. the ratio of girls to boys in a particular class in 3:2 The learner uses unitary method in solving various word problems e.g. if the cost of a dozen notebooks is given she finds the cost of seven notebooks by first finding the cost of 1 notebook.
• Ch-9 Data Handling	**Uncertainty & Data**	The learner arranges given/collected information such as expenditure on different items in a family in the last six months, in the form of table, pictograph and bar graph and interprets them.
• Ch-1 Knowing Our Numbers	**Quantity**	The learner solves problems involving large numbers by applying appropriate operations (addition, subtraction, multiplication and division)
• Ch-2 Whole Numbers	**Quantity**	The learner recognises and appreciates (through patterns) the broad classification of numbers as even, odd, prime, co-prime, etc.

• Ch-3 Playing With Numbers	**Quantity**	The learner applies HCF or LCM in a particular situation.
• Ch-6 Integers	**Quantity**	The learner solves problems involving addition and subtraction of integers.
• Ch-7 Fractions	**Quantity**	The learner uses fractions in different situations which involve money, length, temperature etc. e.g. 7½ metres of cloth etc.
• Ch-8 Decimals	**Quantity**	The learner solves problems on daily life situations involving addition and subtraction of decimals.

CLASS VII

NCERT CHAPTERS	**PISA LITERACY AREA**	**LEARNING OUTCOMES**
• Ch-5 Lines and Angles	**Shape & Space**	The learner classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given. The learner verifies the properties of various pairs of angles formed when a transversal cuts two lines.
• Ch-6 The Triangle and its Properties	**Shape & Space**	The learner finds the unknown angle of a triangle when its two angles are known.

• Ch-7 Congruence of Triangles	**Shape & Space**	The learner explains congruency of triangles on the basis of the information given about them like (SSS, SAS, ASA, RHS)
• Ch-10 Practical Geometry	**Shape & Space**	The learner using a ruler and a pair of compasses constructs a line parallel to a given line from a point outside it and constructs triangles on the basis of congruency.
• Ch-11 Perimeter and Area	**Shape & Space**	The learner finds out the approximate area of closed shapes by using unit square grid/graph sheet and also calculates areas of the regions enclosed in a rectangle and a square.
• Ch-14 Symmetry	**Shape & Space**	The learner differentiates between regular and irregular polygon. The learner understands line symmetry and rotation symmetry
• Ch-15 Visualising Solid Shapes	**Shape & Space**	The learner sees hidden parts of the solid shapes
• Ch-4 Simple Equations	**Change & Relationship**	The learner represents daily life situations in the form of a simple equation and solves it.
• Ch-8 Comparing Quantities	**Change & Relationship**	The learner distinguishes quantities that are in proportion. e.g. tells that 15, 45, 40, 120 are in proportion as 15/45 is the same as 40/120. The learner solves problems related to conversion of percentage to fraction and decimal and vice-versa. The learner calculates profit/loss percent and rate percent in simple interest.

• Ch-12 Algebraic Expressions	**Change & Relationship**	The learner adds/subtracts algebraic expressions.
• Ch-13 Exponents and Powers	**Change & Relationship**	The learner uses exponential form of numbers to simplify problems involving multiplication and division of large numbers.
• Ch-3 Data Handling	**Uncertainty & Data**	The learner finds various representative values for simple data from her/his daily life contexts like mean, median and mode. The learner also recognises variability in real-life situation such as, variations in the height of students in her class and uncertainty in happening of events like throwing a coin. The learner interprets data using bar graph, such as consumption of electricity is more in winters than summer, runs scored by a team in first 10 overs, etc. The learner determines the probability of an event especially in case of 2 dice and cards.
• Ch-1 Integers	**Quantity**	The learner multiplies/divides two integers
• Ch-2 Fractions and Decimals	**Quantity**	The learner interprets the division and multiplication of fractions. The learner uses algorithms to multiply and divide fractions/decimals.
• Ch-9 Rational Numbers	**Quantity**	The learner solves problems related to daily life situations involving rational numbers.

CLASS VIII

NCERT CHAPTERS	PISA LITERACY AREA	LEARNING OUTCOMES
• Ch-3 Understanding Quadrilaterals	**Shape & Space**	The learner solves problems related to angles of a quadrilateral using angle sum property. The learner verifies properties of parallelograms and establishes the relationship between them through reasoning.
• Ch-4 Practical Geometry	**Shape & Space**	The learner constructs different quadrilaterals using compasses and straight edge.
• Ch-10 Visualising Solid Shapes	**Shape & Space**	The learner represents 3D shapes on a plane surface, such as sheet of paper, blackboard, etc. The learner verifies Euler's relation through pattern.
• Ch-11 Mensuration	**Shape & Space**	The learner estimates the area of shapes like trapezium and other polygons by using square grid/graph sheet and verifies using formulas. The learner finds the area of a polygon and also the surface area and volume of cuboidal and cylindrical objects.
• Ch-2 Linear Equations in one Variable	**Change & Relationship**	The learner solves puzzles and daily life problems using variables.
• Ch-8 Comparing Quantities	**Change & Relationship**	The learner applies the concept of percentage in profit and loss situation in finding discount, VAT and compound interest, for example, calculate discount percent when marked price and actual discount are given or find profit per cent when cost price and profit in a transaction are given.

• Ch-9 Algebraic Expression & Identities	**Change & Relationship**	The learner uses various algebraic identities in solving problems of daily life
• Ch-12 Exponents & Powers	**Change & Relationship**	The learner solves problems with integral exponents.
• Ch-14 Factorisation	**Change & Relationship**	The learner multiplies algebraic expressions, for example, expands $(2x-5)(3x^2+7)$.
• Ch-13 Direct & Inverse Proportion	**Change & Relationship**	The learner solves problems based on direct and inverse proportions
• Ch-5 Data Handling	**Uncertainty & Data**	The learner makes hypotheses on chances of future events on the basis of its earlier occurrences or available data like, after repeated throws of dice and coins
• Ch-15 Introduction to Graphs	**Uncertainty & Data**	The learner draws and interprets bar charts and pie charts.
• Ch-1 Rational Numbers	**Quantity**	The learner generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns The learner finds out as many rational numbers as possible between two given rational numbers.
• Ch-6 Squares & Square Roots	**Quantity**	The learner generalises properties of addition, subtraction, multiplication and division of rational numbers through patterns The learner finds out as many rational numbers as possible between two given rational numbers.

• Ch-7 Cubes & Cube Roots	**Quantity**	The learner finds cubes and cube roots of numbers using different methods
• Ch-16 Playing with Numbers	**Quantity**	The learner proves divisibility rules of 2, 3,4, 5, 6, 9 and 11.

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CLASS IX

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NCERT CHAPTERS	PISA LITERACY AREA	LEARNING OUTCOMES
• Ch-5 Euclid's Geometry	**Shape & Space**	The learner derives proof of mathematical statements particularly related to geometrical concepts, like parallel lines, triangles, quadrilaterals, circles etc. by applying axiomatic approach and solves problems using them.
• Ch-6 Lines & Angles	**Shape & Space**	The learner classifies pairs of angles based on their properties as linear, supplementary, complementary, adjacent and vertically opposite and finds value of the one when the other is given. The learner verifies the properties of various pairs of angles formed when a transversal cuts two lines.
• Ch-7 Triangles	**Shape & Space**	The learner explains congruency of triangles on the basis of the information given about them like (SSS, SAS, ASA, RHS)

• Ch-8 Quadrilaterals	**Shape & Space**	The learner differentiates between different types of quadrilaterals like parallelogram, square, rectangle etc. The learner applies the properties of quadrilaterals.
• Ch-9 Area of Parallelogram & Triangles	**Shape & Space**	The learner relates the areas of two parallelogram , triangle and parallelogram etc
• Ch-10 Circles	**Shape & Space**	The learner understands the different parts of a circle , angles in a same segment ,cyclic quadrilaterals etc.
• Ch-11 Constructions	**Shape & Space**	The learner constructs different geometrical shapes like bisectors of line segments, angles, and triangles under given conditions and provides reasons for the processes of such constructions.
• Ch-12 Heron's Formula	**Shape & Space**	The learner finds area of triangles using all types of appropriate formulas.
• Ch-13 Surface Area & Volume	**Shape & Space**	The learner derives formulas for surface areas and volumes of different solid objects like, cubes, cuboids, right circular cylinders/ cones, spheres and hemispheres and applies them to objects found in the surroundings.
• Ch-3 Coordinate Geometry	**Shape & Space**	The learner develops strategies to locate points in a Cartesian plane.
• Ch-4 Linear Equations in two Variables	**Change & Relationship**	The learner relates the algebraic and graphical representations of a linear equation in one/two variables and applies the concepts to daily life situations.

• Ch-2 Polynomials	**Change & Relationship**	The learner identifies or classifies polynomials among algebraic expressions and also factorises them by applying appropriate algebraic identities.
• Ch-14 Statistics	**Uncertainty & Data**	The learner identifies and classifies the daily life situations in which mean, median and mode can be used. The learner analyses data by representing it in different forms like, tabular form (grouped or ungrouped), bar graph, histogram (with equal and varying width and length), and frequency polygon.
• Ch-15 Probability	**Uncertainty & Data**	The learner calculates empirical probability through experiments.
• Ch-1 Number System	**Quantity**	The learner applies logical reasoning in classifying real numbers, proving their properties and using them in different situations.

CLASS X

NCERT CHAPTERS	PISA LITERACY AREA	**LEARNING OUTCOMES**
• Ch-6 Triangles	**Shape & Space**	The learner works out ways to differentiate between congruent and similar figures. The learner establishes properties for similarity of two triangles logically using different geometrical criteria established earlier such as Basic Proportionality Theorem etc.

• Ch-7 Coordinate Geometry	**Shape & Space**	The learner derives formulae to establish relations for geometrical shapes in the context of a coordinate plane, such as finding the distance between two given points, to determine coordinates of a point between any two given points, to find area of a triangle etc.
• Ch-10 Circles	**Shape & Space**	The learner understands different concepts of tangents, secants and circles.
• Ch-11 Constructions	**Shape & Space**	The learner constructs • a triangle similar to a given triangle as per a given scale factor. • a pair of tangents from an external point to a circle and justify procedures. The learner examines the steps of geometrical constructions and reasons out each step.
• Ch-12 Areas related to Circles	**Shape & Space**	The learner determines areas of various 2D objects, designs around them e.g. design on a handkerchief, design of tiles on the floor, geometry box etc.
• Ch-13 Surface Area & Volume	**Shape & Space**	The learner finds surface areas and volumes of 3D objects in the surroundings by visualising them as a combination of different solids like cylinder and a cone, cylinder and a hemisphere, combination of different cubes etc.
• Ch-13 Linear Equations in two Variables	**Change & Relationship**	The learner finds solutions of pairs of linear equations in two variables using graphical and different algebraic methods.
• Ch-2 Polynomials	**Change & Relationship**	The learner develops a relationship between algebraic and graphical methods of finding the zeroes of a polynomial.
• Ch-8 Introduction to Trigonometry	**Change & Relationship**	The learner determines all trigonometric ratios with respect to a given acute angle (of a right triangle)

• Ch-9 Applications of Trigonometry	**Change & Relationship**	The learner uses trigonometric ratios in solving problems in daily life contexts like finding heights of different structures or distances between them.
• Ch-4 Quadratic Equations	**Change & Relationship**	The learner demonstrates strategies of finding roots and determining the nature of roots of a quadratic equation.
• Ch-14 Statistics	**Uncertainty & Data**	The learner calculates mean, median and mode for different sets of data related with real-life contexts and observe graphical data to find mode, median etc.
• Ch-15 Probability	**Uncertainty & Data**	The learner determines the probability of an event.
• Ch-1 Real Numbers	**Quantity**	The learner generalises properties of numbers and relations among them studied earlier to evolve results, such as, Euclid's division algorithm, Fundamental Theorem of Arithmetic, and applies them to solve problems related to real life contexts.
• Ch-5 Arithmetic Progression	**Quantity**	The learner develops strategies to apply the concept of A.P to daily life situations.

Note to Teacher:

Mathematics teachers may collaborate with other subject teachers to find out the linkage between the subjects. Teachers must do the content analysis for each unit to identify the processes involved, content areas, learning objectives etc. to give inputs on mathematisation. This should be the basis of their daily lesson plan.

6. ASSESSMENT AS A TOOL

From the earlier discussion it is clear that mathematical literacy can be achieved only if we give opportunities to our students for mathematization and using Mathematics in their day to life. To achieve this, conscious efforts must be taken right from the primary classes. All the concepts in Mathematics be introduced by linking the same with real life. Children should be convinced about the utility of learning Mathematics in day to day life. Hence it is important that we make use of Assessment as a tool for learning.

In the majority of the cases we assess at the end of a task, unit of work and end of the academic year. Assessment of learning assists teachers in using evidence of student learning to assess achievement against outcomes and standards. Sometimes referred to as 'summative assessment', it usually occurs at defined key points during a unit of work or at the end of a unit, term or semester, and may be used to rank or grade students. Most of our teaching focuses on getting better results in the year-end examination , especially Board examinations at the end of class X and XII. Though year-end examinations give an indication of the learning levels, teachers must see that variety of assessments are used during their classroom transactions. Most of the time use of Assessment as a tool 'for' and 'as' learning is neglected.

6.1   Assessment for Learning:

'Assessment for learning' is the process of seeking and interpreting evidence for use by learners and their teachers to decide where the learners are in their learning, where they need to go and how best to get there. Assessment for Learning is also known as formative assessment. Activities associated with summative assessment (Assessment of Learning) result in an evaluation of student achievement - for example, allocation to a level or standard or allocation of a letter or numerical grade, which might later appear in a report. Activities associated with formative assessment (Assessment for Learning) do not result in an evaluation. Information about what a student knows, understands and is able to do is used by both the teacher and the learner to determine where learners are in their learning and how to achieve learning goals. However it must kept in mind that Assessment of Learning can also be considered as Assessment for learning, if we are giving inputs to students based on their performance.

6.2   Strategies for Learning from Assessment :

There can be number of classroom strategies that are particularly effective in promoting formative assessment practice. **Some of the 'Assessment for Learning' strategies are:**

1)   **Strategic use of Questioning :**

    Questioning is used not only as a pedagogical tool but also as a deliberate way for the teacher to find out what students know, understand and are able to do.

2)   **Effective feedback from teachers :**

    Effective teacher feedback focuses on telling the students what they have achieved and where they need to improve. Importantly, the feedback provides specific suggestions about how that improvement might be achieved. Such a feedback can be given during the classroom teaching-learning process as well. There is a scope for teachers to give valuable inputs for improvement after assessing the answer scripts.

3) Feedback from Peers :

Peer feedback occurs when a student tells another student what they have achieved and where improvement is necessary. Again, the feedback provides specific suggestions to help achieve improvement.

4) Self Assessment by students :

Student self-assessment encourages students to take responsibility for their own learning. It incorporates self-monitoring, self-assessment and self-evaluation. Students may be asked to prepare questions themselves. The process of preparation of questions by students help them understand their own level of learning

5) Formative use of Summative Assessment :

Summative assessment is a necessary aspect of education. Formative use can be made of summative assessment, both before and after the assessment. Item analysis of summative assessment can help in identifying the area of focus in the next session or next class. Individual feedback can be given for better performance in the future.

6.3 Assessment as Learning:

It is important to understand that Assessment can also be used 'as learning'. Assessment as learning occurs when students are their own assessors. Students monitor their own learning, ask questions and use a range of strategies to decide what they know and can do, and how to use assessment for new learning. Through this process students are able to learn about themselves as learners and become aware of how they learn – become metacognitive (knowledge of one's own thought processes). Students will be able to reflect on the following during any assessment.

➢ What is the purpose of learning these concepts and skills?

➢ What do I know about this topic?

➢ What strategies do I know that will help me learn this?

➢ Am I understanding these concepts?

➢ What are the criteria for improving my work?

➢ Have I accomplished the goals I set for myself?

Though assessment of learning and assessment as learning are important, assessment for learning needs to be focused. Assessment thus can be used as an effective tool for learning. It is important that teachers should also be able to develop Assessment tools which will help in learning.

In conclusion PISA may be considered as an opportunity to the subject teachers to improve the quality of classroom transactions. It may also be ensured that Assessment is the Integral part of curriculum transitions at all levels and must be used "AS, OF and FOR" learning.

6.4 Art of making questions :

Questioning is an art. The quality of questions asked by the teacher and student decides the quality of assessments and thereby quality of learning. A "Good" questions can set the stage for meaningful classroom discussion and learning. It invites students

➢ to think,

➢ to understand, and

➢ to share their mathematical journey

Questions should NOT to be used to terrorise students; they should help students to make sense of the Mathematics. An open-ended approach of question will empower students to unravel their misconceptions. Development of such questions not only requires the application of facts and procedures but encourages students to make connections and generalizations. It is important that the questions are accessible to all students in their language and offer an entry point for all students. The answers to good questions will lead students to wonder more about a topic and to perhaps construct new questions themselves. Good questions are used for :

➢ problem-solving,

➢ application of concepts and procedures, and

➢ the development of a variety of thinking skills

While constructing good questions, it is essential for teachers to understand the Mathematics embedded in the question. Questions must be presented logically using accessible mathematical language to set clear and reasonable expectations for student work. Questions should have scope for individual approaches, methods, and/or answers. Teachers can add variety or more data to a question to ensure accessibility for all students. Teachers may make good use of concrete materials and give ample time to learners for discovery and consolidation of answers, strategies, and problem solving. Teachers should use a variety of forms for asking questions in the classroom as a diagnostic tool. Individual attention may be given to each student with proper remedial measures based on their responses. Few examples of good questions are given below:

1. Always, Sometimes or Never
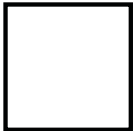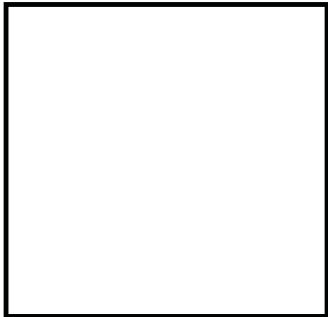
Multiple of 5 end in 5. Always / Sometimes / Never	Justify your answer.

Multiple of 2 end in an odd number. Always / Sometimes / Never	Justify your answer.

Multiple of 7 are odd number. Always / Sometimes / Never	Justify your answer.

2. Probe and Response

Square 1

x Units

Square 2

2x Units

Tick the statement that describes how the area of square 2 compares with the area of square 1:

A. doubles in size
B. same size
C. four times in size
D. not enough information to compare

3. Create the problem

Here is a computational equation 2/3 of 15=10. Create three different problems that can be solved using this equation.

Problem 1

Problem 2

Problem 3

4. Examples and Non examples

Shapes with four sides that are rectangles.	Shapes with four sides that are not rectangles.
Draw as many different examples as you can here.	Draw as many different examples as you can here.

State the rule you used to decide whether a four- sided shape is a rectangle.

5. Question to Reason

This is an isosceles triangle. Why is this an isosceles triangle? What are the possible integral values of angles for which it is possible to construct the isosceles triangle?
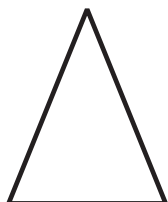
Look at the example $(-2) \times (-3) = 6$

Why multiplying two negative numbers results in a positive number?

Describe the following algebraic expression:

$3x^3 + 4x$

6. Make your own group
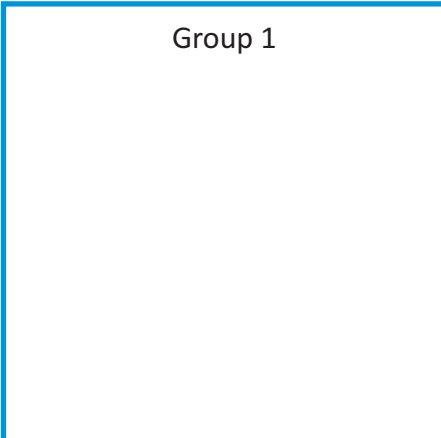
Here is the list of terms associated with Algebra:

Variable, constant, equation, equal, rational, number, expression, integer, coefficient, equation, solution, LHS, RHS
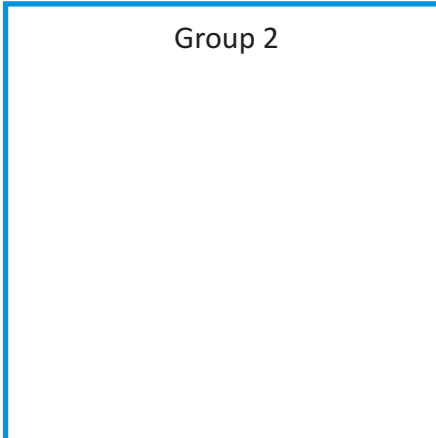
Organize them in different groups under following conditions:
- There must be at least three groups.
- Each group must contain at least three terms.
- Terms can be placed in more than one group.
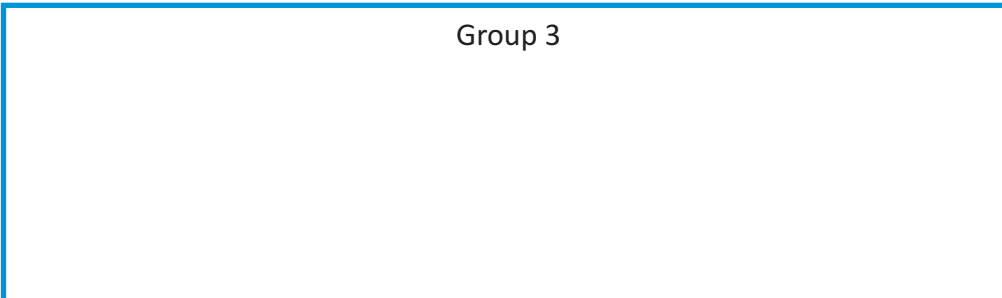
Group 1	Group 2

Group 3

NOTE TO TEACHER:

Assessment should be inbuilt in the teaching-learning process. Mathematics teachers must prepare lesson plans with the Assessment strategies which will help children learn Mathematics. The Unit tests and Final examination responses also may be analyzed and individual feedback may be given to the students for improvement. Students must be given a chance to reflect on their responses and their performance in the assessment.

7. SAMPLE QUESTIONS

7.1 PISA Sample Questions

A. CUBES (8)

In this photograph you see six dice, labeled (a) to (f). For all dice there is a rule:
The total number of dots on two opposite faces of each dice is always seven.
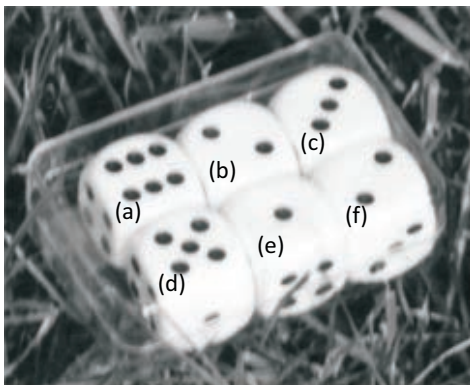
Question 1: CUBES   M145Q01

Write in each box the number of dots on the bottom face of the dice corresponding to the photograph

(a)	(b)	(c)
(d)	(e)	(f)

B. GROWING UP (8)

Youth grows taller

In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.
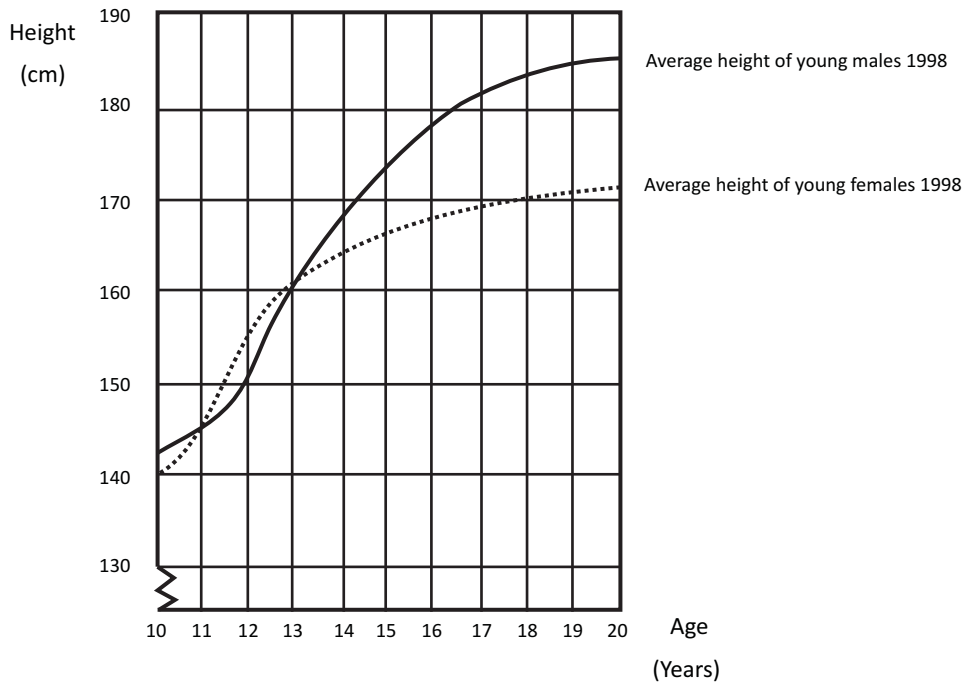
Question 1: GROWING UP   M150Q01

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

....Cm

Question 2: GROWING UP   M150Q02

Explain how the graph shows that, on average the growth rate for girls slows down after 12 years of age.

....
....
....

Question 3: GROWING UP   M150Q03

According to this graph, on average, during which period in their life are females taller than males of the same age?

....
....
....

C. WALKING (8)

The picture shows the footprints of a man walking. The pacelength P is the distance between the rears of two consecutive footprints.

For men, the formula, $\frac{n}{p} = 140$, gives an approximate relationship between n and P where,

n = number of steps per minute, and

P = pacelength in meters

Question 1: WALKING   M124Q01

If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pace length? Show your work.

....

....

....

Question 2: **WALKING**   M124Q03

Bernard knows his pacelength is 0.80 meters. The formula applies to Bernard's walking. Calculate Bernard's walking speed in meters per minute and in kilometers per hour. Show your work.

....

....

....

D. ROBBERIES (8)

A TV reporter showed this graph and said:

"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999.
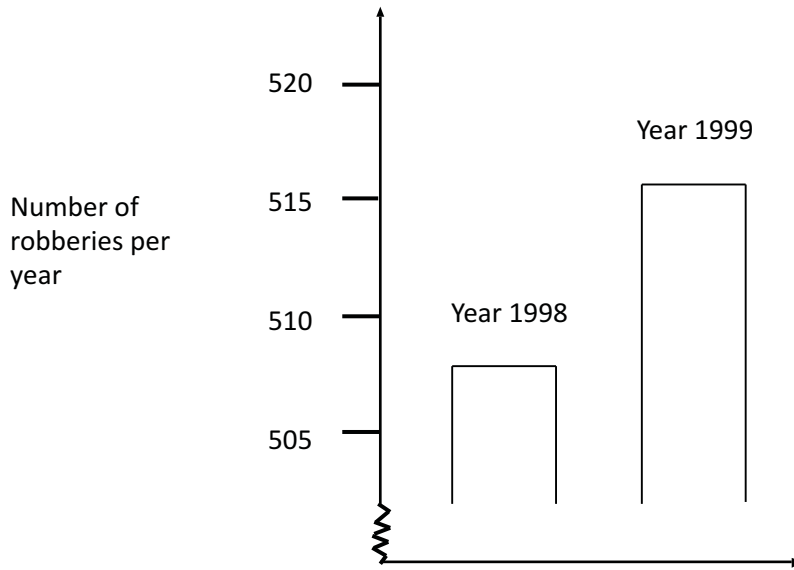
Question 1: ROBBERIES   M179Q01

Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give explanation to support your answer.

....

....

....

....

....

....

....

....

....

E. CARPENTER (8)

A carpenter has 32 meters of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.

A

6 m

10 m

B

6 m

10 m

C

6 m

10 m

D

6 m

10 m
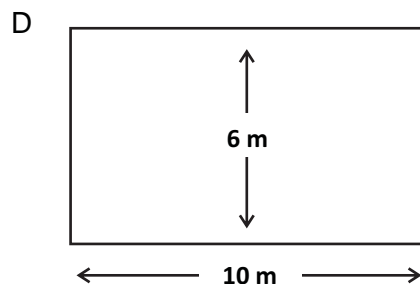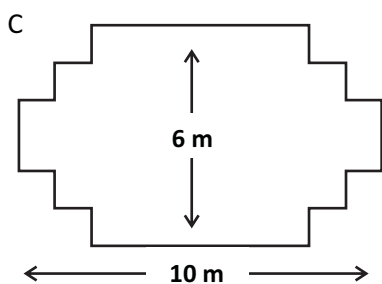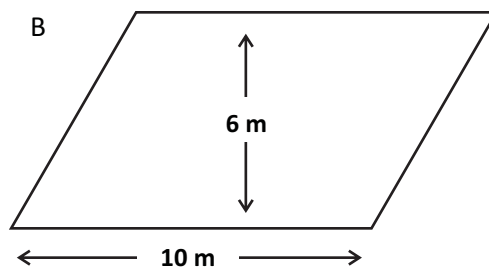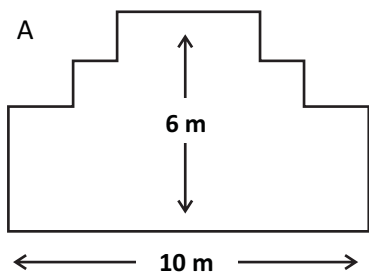
Question 1: CARPENTER   M266Q01

Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 meters of timber.

Garden bed	Using this design, can the garden bed be made with
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

F. EXPORTS (8)

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.

Total annual exports from Zedland in millions of zeds, 1996 to 2000
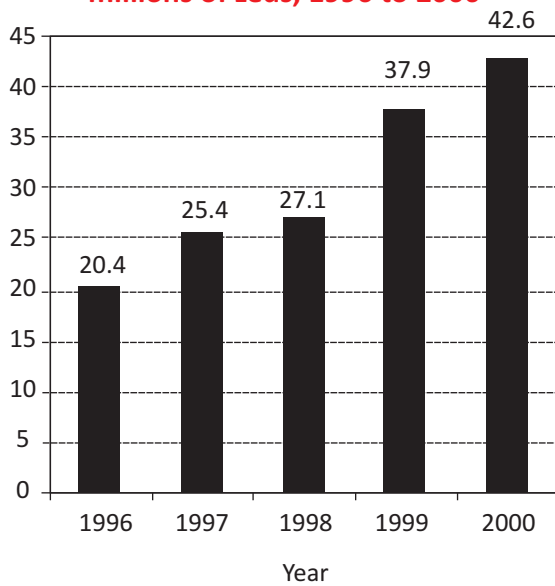
Distribution of exports from Zedland in 2000
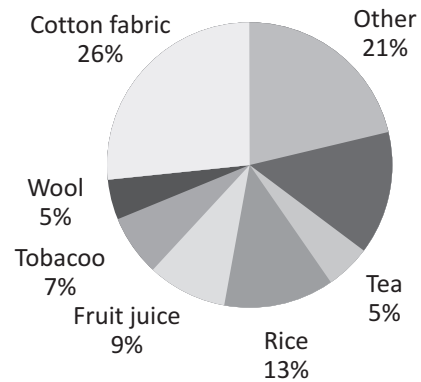
Question 1: EXPORTS   M438Q01 - 019

What was the total value (in millions of zeds) of exports from Zedland in 1998?

Answer: ....

Question 2 : EXPORTS   M438Q02

What was the value of fruit juice exported from Zedland in 2000?

A   1.8 million zeds.
B   2.3 million zeds.
C   2.4 million zeds.
D   3.4 million zeds.
E   3.8 million zeds.

G. SKATEBOARD (8)

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are

Product	Price in zeds	
Complete Skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Question 1: SKATEBOARD   M520Q01aM520Q01b

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: .... zeds.

(b) Maximum price: .... zeds.

Question 2: SKATEBOARD   M520Q02

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

    A.   6

    B.   8

    C.   10

    D.   12

Question 3: SKATEBOARD   M520Q03

Eric has 120 zeds to spend and wants to buy the most expensive skateboard he can afford.
How much money can Eric afford to spend on each of the 4 parts? Put your answer in the table below.

Part	Amount (zeds)
Deck	
Wheels	
Trucks	
Hardware	

7.2 Other Sample Questions

A. RAIL TIME TABLE

The Table below gives the 2013 Monsoon Timetable for The Konkanhanya Mandovi, Jan Shatabdi, Matsyagandha, CSTM Superfast, Mangala and Netravati Expresses. The old timings or regular non-monsoon timings are given side by side for comparison. The table only gives information about important stops

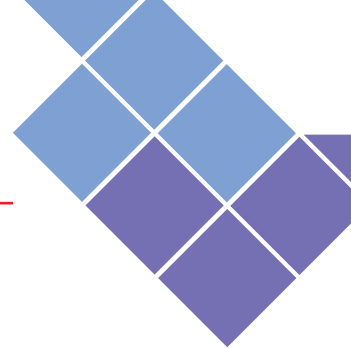| Train		10112 KONKAN KANYA		10104 MANDOVI EXPRESS		12052 JAN SHATABDI		12620 MATSYA GANDHA		12134 CSTM SUPERFAST		12617 MANGALA		16346 NETRAVATI	
Timing		Regular	Mansoon	Regular	Mansoon	Regular	Mansoon	Regular	Mansoon	Regular	Mansoon	Regular	Mansoon	Regular	Mansoon
TVC	TRIVANDRUM CNTL													9:50	9:50
ERS	ERNAKULAM JN											13:15	10:45	14:05	14:05
TCR	THRISUR											14:38	12:05	15:38	15:38
SRR	SHIRANUR JN											15:35	13:05	16:55	16:55
CLT	KOZHIKODE											17:30	15:05	19:00	19:00
CAN	KANNUR											19:15	16:55	20:40	20:40
MAJN	MANGALORE JN									14:00	16:45	22:10	19:50	23:30	23:30
MAQ	MANGALORE CTRL							14:35	12:50						
SL	SURATHKAL							15:16	13:36	14:32	17:8			0:24	0:22
UD	UDUPI							15:58	14:22	15:07	17:50	23:16	21:08	1:02	1:02
KUDA	KUNDAPURA							16:8	14:54			23:38	21:36	1:30	1:34
BYNR	MOOKAMBIKA RD							17:02	15:27	15:47	18:47			2:00	2:08
BTJL	BHATKAL							17:20	15:46			0:18	22:24	2:20	2:28
MEDW	MURDESHWAR							17:40	16:02					2:40	2:52
KT	KUMTA							18:20	16:42	16:42	19:42			3:12	3:32
KAWR	KARWAR							19:24	17:47	17:42	20:34	1:56	0:07	4:02	4:38
MAO	MADGAON	18:00	16:45	9:30	8:30	14:30	12:10	20:0	19:15	18:50	21:50	3:20	1:35	5:10	5:55
KRMI	KARMALI	18:34	17:8	10:04	9:10									5:48	6:26
THVM	THIVIM	18:56	17:36	10:26	9:30	15:16	12:54					4:10	2:10	6:06	6:48
SWV	SAWANTWADI RD	19:38	18:16	11:02	10:07										
KUDL	KUDAL	20:02	18:48	11:20	10:30	15:58	13:56	22:20	20:54					7:08	8:02
SNDD	SINDHUDURG	20:14	18:58	11:34	10:42										
KKW	KANKAVALI	20:40	19:14	11:54	11:02	16:20	14:22					0:04	5:44	3:34	
RN	RATNAGIRI	23:05	22:05	14:25	13:35	17:50	16:25	0:30	23:15	20:30	2:20	7:40	6:10	9:20	11:10
SGR	SANGMESHWAR	23:40	22:46	15:04	14:6					22:20					
CHI	CHIPLUN	0:12	23:50	15:40	15:0	18:45	17:50	1:38	0:56			8:40	8:00	11:12	12:50
KHED	KHED	0:42	0:28	16:02	16:02										
MNI	MANGAON	1:45	2:02	17:10	17:20									13:45	15:25
PNVL	PANVEL	4:10	4:10	19:30	19:30			5:15	5:15	2:40	8:5	13:00	13:00	15:15	16:45
TNA	THANE	4:55	4:55	20:30	20:30	22:35	22:35	5:55	5:55	3:30	9:25			16:05	17:35
DR	DADAR	5:20	5:20	21:03	21:03	23:05	23:05							16:40	18:04
CSTM	MUMBAI CST	5:50	5:20	21:40	21:40					4:25	10:33				
LTT	LOKMANYA TILAK T							6:35	6:35						

(Source : www.24coaches.com/konkan.railways-mansoon-)

Question 1: RAIL TIME TABLE

Which train is the slowest as per train schedule given above? Justify your answer.

....

Question 2: RAIL TIME TABLE

Out of Konkan Kanya and Mandvi express , which has least delay due to monsoon. Justify your answer
....

Question 3: RAIL TIME TABLE

Due to bad weather in monsoon, Konkan Kanya departed 2 hours late from its Monsoon scheduled time, how much speed should it increase to reach Mumbai CST as per scheduled time of arrival. The distance between Madgaon station and Mumbai CST is approximately 600 km

....

Question 4: RAIL TIME TABLE

How much more time will it take, if you travel by 12052 - Jan Shatabdi from Kudal to Chiplun during monsoon as compared to any other season.

....

Question 5: RAIL TIME TABLE

If distance between Madgaon Station and Mumbai CST is 600 kms , what is the approximate average speed at which 12134- CSTM superfast moves on a regular day?

....

B. PACKING OF MANGOES

For domestic market, usually mangoes are packed in wooden boxes, details of which are given below

Type of Carton	Inner Dimension (cm^3)	Capacity (kg)
Wooden crates	45X30X30 (Ratnagiri)	16-18
	21.6X21.6X42 (Malihabad)	10-11

If we pack 'Ratnagiri' mangoes as shown in the picture below, with three layers and the weight of the mangoes packed in a wooden crate comes out to be 18 kg weight then answer the following questions.

(Source : www.agriexchange.apeda.gov.in)

Question 1 : PACKING OF MANGOES

Find out approximate weight of each mango. If 10 % of space is used by packing material and free space, find out the approximate volume of each mango.

………..

Question 2 : PACKING OF MANGOES

What is the approximate range of weight of one layer of mangoes from the data given?

……..

Question 3 : PACKING OF MANGOES

One wooden crate full of Ratnagiri mangoes costs Rs 360. If one-third of the mangoes are found to be rotten and are discarded , find out at what rate ,the remaining mangoes should be sold so that the shop keeper recovers his/her cost ?

……..

Question 4 : PACKING OF MANGOES

If there are 960 children in a school , how many crates should be bought, so that each child gets one mango?

……..

<center>**C. BOTTLED WATER AND ENVIRONMENT**</center>

<center>(Source : www.watertogo.ey)</center>

Read the above poster and answer the question given below:

Question 1 : BOTTLED WATER AND ENVIRONMENT

On an average, how much oil is used in manufacturing and shipping the number of bottles consumed annually in US and Europe as per approximate estimates given above?

....

Question 2 : BOTTLED WATER AND ENVIRONMENT

How much more, one has to spend for one litre of water as compared to one liter of petrol?

....

Question 3 : BOTTLED WATER AND ENVIRONMENT

 If one Euro is equal to Rs 76 , then how much approximate total amount will be spend in rupees for annual consumptions of bottles by US and Europe?
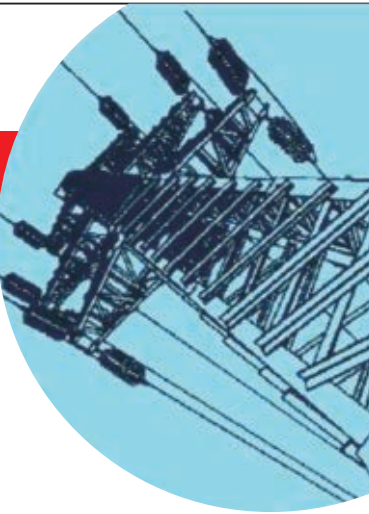
....

D. ELECTRICITY TARIFF IN MUMBAI

The Maharashtra State Electricity Distribution Company Limited (MSEDCL), which supplies power to over 2.5 crore residential, commercial, and industrial consumers increased the electricity tariff by 6% for every unit. The details of the increased power tariffs as on 1ˢᵗ April 2019 by various other Companies and the percentage change in tariffs are given below:

MERC's tariff hike for power distributors from April 1, 2019

Distributor	Adani Electricity	Tata Power	MSEDCL	BEST
Tariff hike (%)	1% ⬆	2% ⬆	6% ⬆	(-3)% ⬇
Units				
0-100	₹4.77	₹4.53	₹4.33	₹2.93
101-300	₹7.90	₹7.66	₹8.23	₹5.18
301-500	₹9.08	₹12.16	₹11.18	₹7.79
501 & above	₹10.92	₹14.74	₹12.78	₹9.20

(Source : www.thehindu.com/news/cities/mumbai)

Based on the information given above, answer the following questions:

Question 1: ELECTRICITY TARIFF IN MUMBAI

What was the tariff of BEST company for 0-100 units prior to April 1, 2019?

....

Question 2: ELECTRICITY TARIFF IN MUMBAI

If you have consumed 250 units in the month of May 2019 with Tatapower connection , what will be your bill amount ?
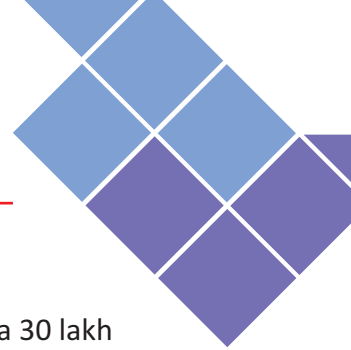
E. INTEREST PAYMENT

The table below shows the breakup of interest and principal payment for each year for a 30 lakh loan for 20 years tenure assuming interest @10%.

Year Loan Breakup for Rs. 30 lakh loan for 20 yrs tenure @10% inrerest				
Year	EMI Yearly (Rs.)	Principal (Rs.)	Interest (Rs.)	Loan Outstanding(Rs.)
1	347408	49642	297766	2950358
2	347408	54841	292567	2895517
3	347408	60583	286825	2834934
4	347408	66927	280481	2768007
5	347408	73935	273473	2694072
6	347408	81677	265731	2612395
7	347408	90229	257179	2522166
8	347408	99678	247730	2422488
9	347408	110116	237292	2312372
10	347408	121645	225763	2190727
11	347408	134384	213024	2056343
12	347408	148456	198952	1907887
13	347408	164000	183408	1743887
14	347408	181174	166234	1562713
15	347408	200145	147263	1362568
16	347408	221103	126305	1141465
17	347408	244255	103153	897210
18	347408	269832	77576	627378
19	347408	298086	49322	329292
20	347408	329292	18116	0

Question 1 : INTEREST PAYMENT

In which year, the difference between Principal and Interest part of EMI is minimum?

....

Question 2 : INTEREST PAYMENT

From which year onwards the Principal part of EMI exceeds the interest part?

....

Question 3 : INTEREST PAYMENT

In the graph chose a suitable scale to show the interest/principal amount as per data given above.
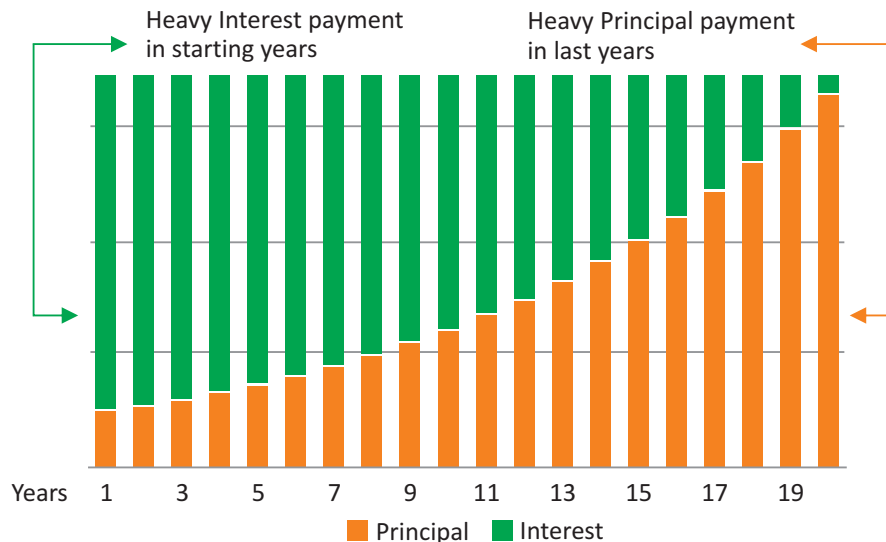
(Source : www.jagoinvestor.com)

F. METRO PARKING RATES

Raj and Shyam stay in the same colony and are colleagues at Indraprastha Gas Limited, Indraprastha, Delhi. But their shifts are different. Raj's office timing is from 9.00 a.m to 5.00 p.m. and Shyam's is 5.00 p.m. to 1.00 a.m. They go to Vaishali metro station daily by car and use the Metro station parking and take Metro to reach their office. Average time of travel to their office is one hour. Based on the revised parking rates at metro stations in Delhi NCR given below, answer the questions given.

Type of vehicle	Four Wheelers (Cars etc.)		Two Wheelers (Scooter, Bike etc.)		Cycle	
	Existing Rates effective Since 1.1.13 (In Rs.)	Revised Rates w.e.f 1.5.18 (In Rs.)	Existing Rates effective Since 1.1.13 (In Rs.)	Revised Rates w.e.f 1.5.18 (In Rs.)	Existing Rates effective Since 1.1.13 (In Rs.)	Revised Rates w.e.f 1.5.18 (In Rs.)
Upto 6 Hours	20	30	10	15	3	5
Upto 12 Hours	30	50	15	25	4	5
More than 12 Hours	40	60	20	30	5	10
Monthly Charges	1,000	1,200	475	600	45	70
Night Charges (Extra)(00:00 Hrs to 5:00 Hrs) Daily	40	60	20	30	5	10
Night Charges (Extra)(00:00 Hrs to 5:00 Hrs) Monthly	1,000	1,200	475	600	45	70

(Source: www.delhimetrorail.com)

Question 1. What is the expense of car parking to Shyam for attending the office in a week (five days) by paying daily charges?

....

Question 2. What is the expense of car parking for Raj for attending office for three days, if he pays daily?

....

Question 3. How much will Raj save, if he uses a monthly pass for car parking in comparison to daily payment, for a five days week? Will your answer be different for different months? Justify your answer.

....

Question 4. If Raj uses a two wheeler instead of a car, how much money will he save in a day?

....

Question 5. Compare the expenses on the monthly pass to Shyam before and after the revision of rates.

....

Question 6. If the parking space meant for a car can accommodate 10 bicycles. What percentage increase /decrease in collection will be there for a single parking area, if a car is replaced by bicycles?

....

Question 7. Compare the percentage increase in parking rates for the duration of 6 hours after revision of rates for cars, two-wheelers and cars .

....

Question 8. Find out the difference in amount of daily pass if used for 12 hours with monthly pass for a scooter.

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8. TEACHERS, THE REAL GAME CHANGERS!

Preparing to benchmark our students on PISA 2021. Certainly a big task but not daunting. With the country's commitment to PISA and students of Chandigarh/KVS/NVS being a sample, you are expected to participate in a series of training programmes and workshops and reorient yourself. You are also expected to prepare tools/ tests for ongoing assessments, engage more with students, assess them more frequently and authentically, and analyze their work periodically. Some may consider this as too much work . . . It is natural and you may feel exasperated at times.

However when you look at it closely you would find, that it is simply a question of realigning. You already spend a lot of time planning your teaching-learning strategies, developing resources and assessing your learners. The high performance of students of schools of Union Territory of Chandigarh, Kendriya Vidyalayas and Navodaya Vidyalayas students in Board examination is a testimony to the sincere efforts that you teachers have been putting in your work. However, we must also acknowledge that there is scope for a lot of improvement in our educational system. Making learning joyful is a precursor of making exams stress free.

Children often feel that examinations are stressful. We thus, need to shift the focus on the way we teach and assess, so that our learners enjoy learning and are also better prepared to meet the challenges of tomorrow's world. We must accept the fact that our learners need new kinds of learning, new skills and new knowledge to thrive in the globally and digitally interconnected world. The gap between the skills students learn in classrooms and the skills student need to participate fully in society is becoming more obvious. We need to focus on providing students with broad sets of skills such as problem-solving, decision making, critical thinking and communication- the 21st Century competencies and skills. We also endorse that opportunities must be established for learners to apply these skills in authentic situations that address real-word problems and issues across their community.

An assessment like PISA focuses on young people's ability to use their knowledge and skills to meet real life challenges rather than merely on the extent to which they have mastered a specific school curriculum. It certainly should be the aim to which we should align. As we share these sentiments, as a teacher, it becomes our important responsibility to reflect on our teaching, learning and assessment practices so as to bring a transformational change in our students' learning experiences. We need to align our education system/curriculum, standards and assessment to global standards and prepare our students for such kinds of assessments that promote learning. It is time to adopt internationally accepted benchmarks and best practices for academic excellence. The onus is on us to set actionable targets in consonance with the paradigm shift in learning that advocates inquiry-based and student-directed learning.
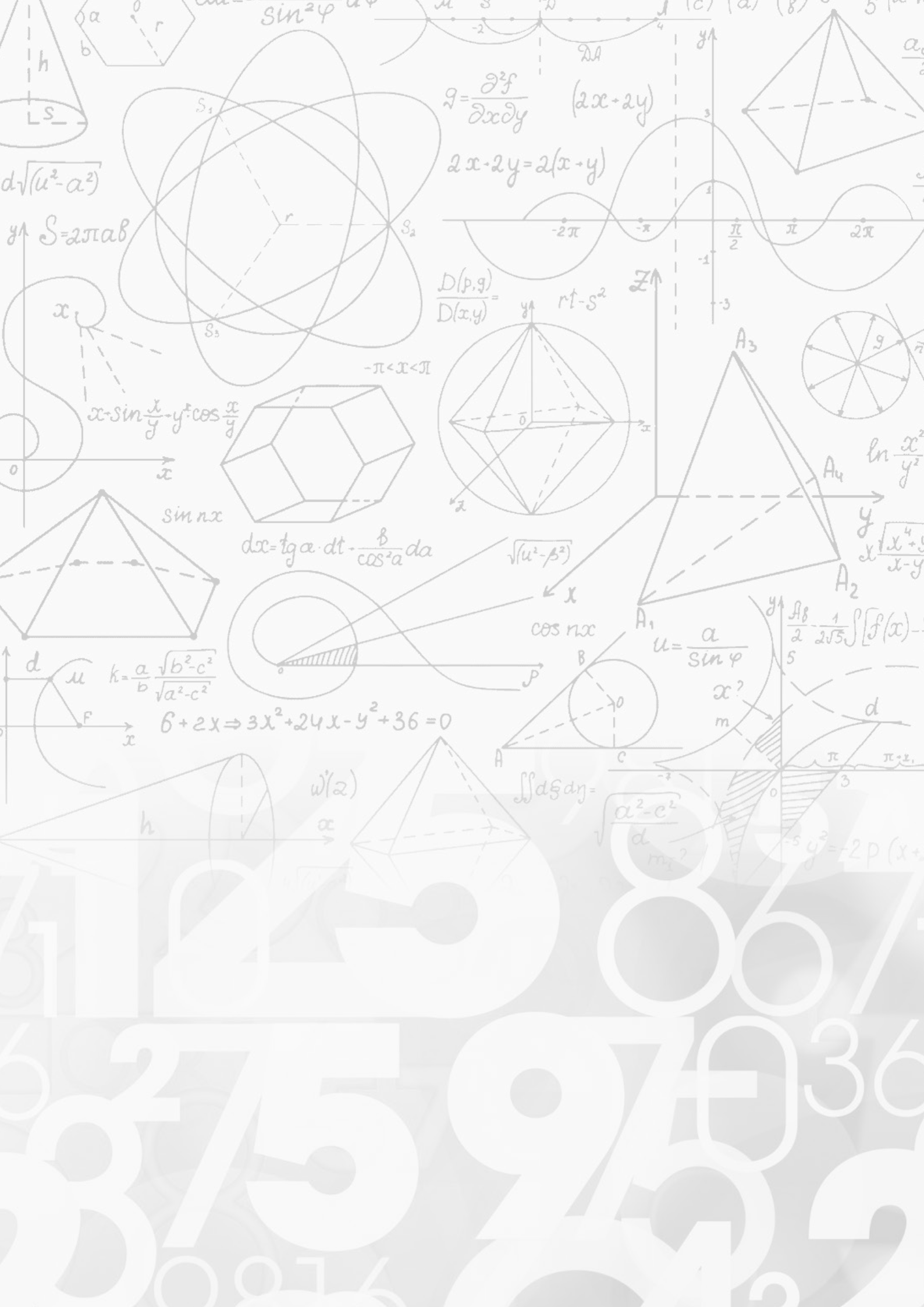
As teachers, we need to bridge this gap between our aspiration for future-ready learning and day to day classroom practices. As a teacher, you can to make a difference and empower your students; you can help them reap benefit from an education that supports them in becoming holistically developed individuals; flourish into healthy, bright and contributing citizens of the 21st century.

And in a nutshell, you are the one who will propel your students to flourish into healthy, bright and contributing citizens of the 21st century. So, let's gear up and be ready to be the game changers!

The future of the world is in my classroom today. -Ivan Welton Fitzwater

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